EE 435

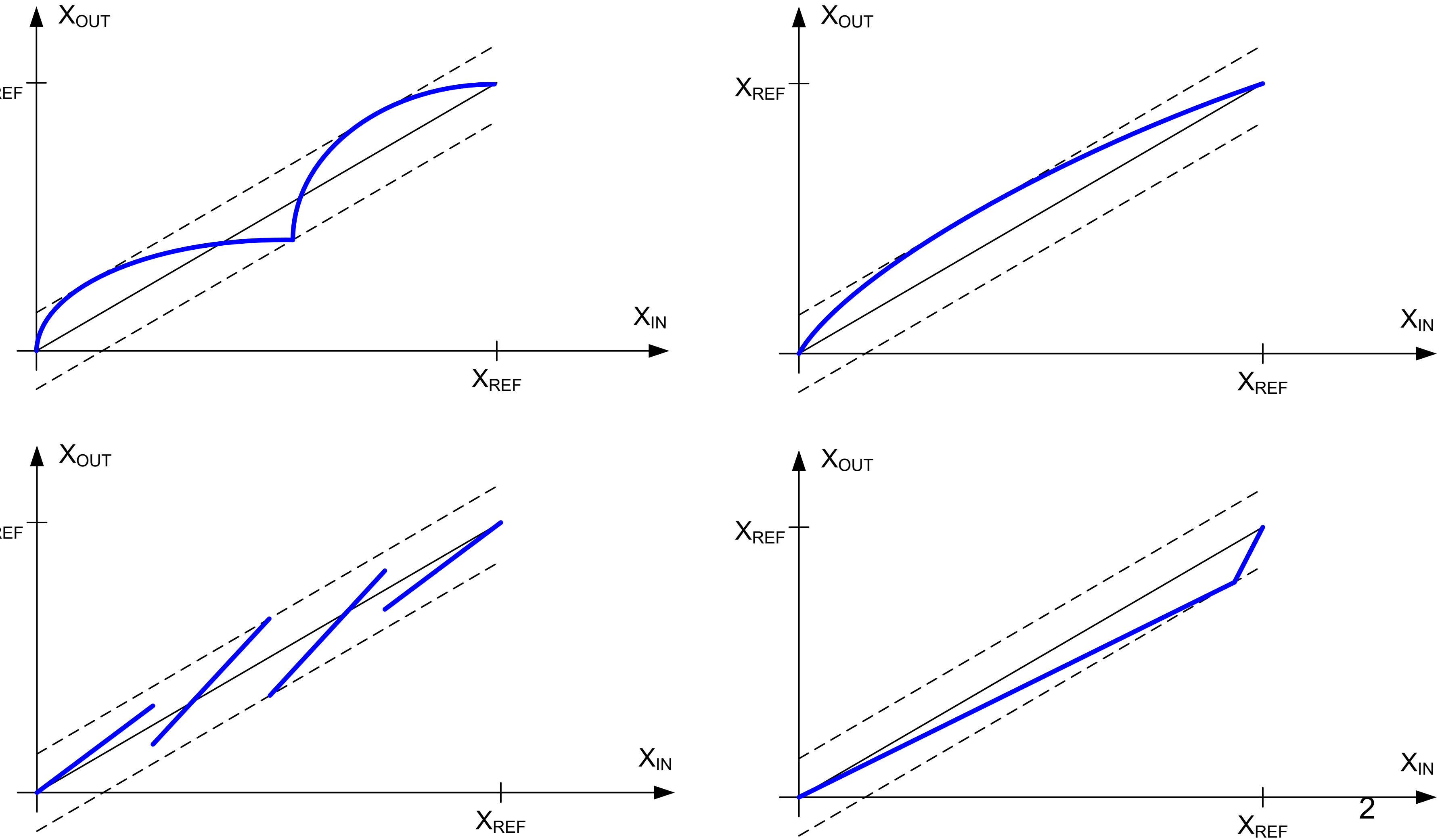
Lecture 29

Data Converters

- Spectral Performance
    - Windowing
- Quantization Noise

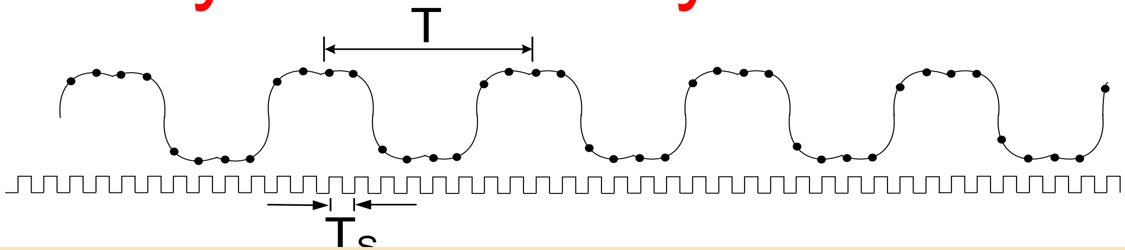
INL Often Not a Good Measure of Linearity

Four identical INL with dramatically different linearity



Review from last lecture

Why is this a Key Theorem?



Review from last lecture

THEOREM: Consider a periodic signal with period T=1/f and sampling period T_S=1/f_S. If N_P is an integer and x(t) is band limited to f_{MAX}, then

$$\left|A_m\right| = \frac{2}{N}\left|X\left(mN_P+1\right)\right| \qquad 0 \le m \le h-1$$

and $X(k)=0$ for all k not defined above

where $\left\langle X(k)\right\rangle_{k=0}^{N-1}$ is the DFT of the sequence $\left\langle x(kT_S)\right\rangle_{k=0}^{N-1}$

N=number of samples, N_P is the number of periods, and h = Int$\left(\frac{f_{MAX}}{f}-\frac{1}{N_P}\right)$

- DFT requires dramatically less computation time than the integrals for obtaining Fourier Series coefficients
- Can easily determine the sampling rate (often termed the Nyquist rate) to satisfy the band limited part of the theorem

Distortion Analysis

How are spectral components determined?

By integral

$$A_k = \frac{1}{\omega T}\left(\int_{t_1}^{t_1+T} f(t)e^{-jk\omega t}dt + \int_{t_1}^{t_1+T} f(t)e^{jk\omega t}dt\right)$$

or

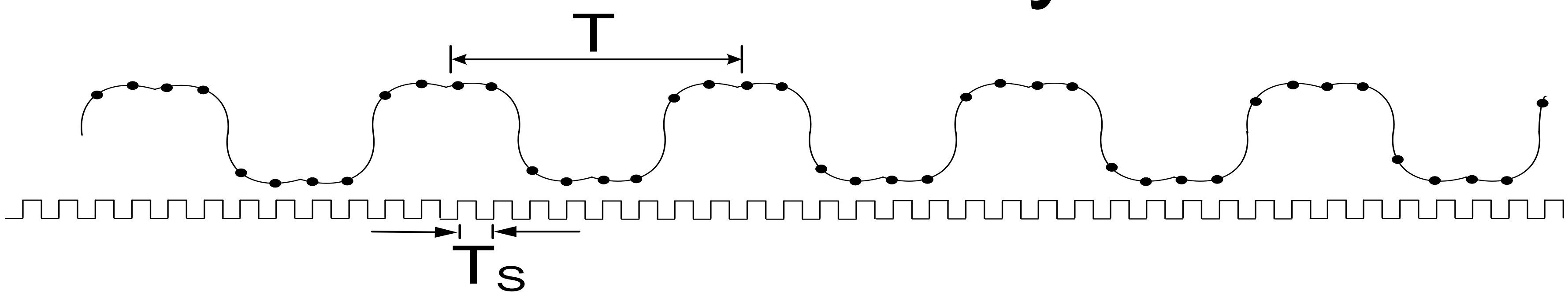
$$a_k = \frac{2}{\omega T}\int_{t_1}^{t_1+T} f(t)\sin(kt\omega)dt \qquad b_k = \frac{2}{\omega T}\int_{t_1}^{t_1+T} f(t)\cos(kt\omega)dt$$

Integral is very time consuming, particularly if large number of components are required

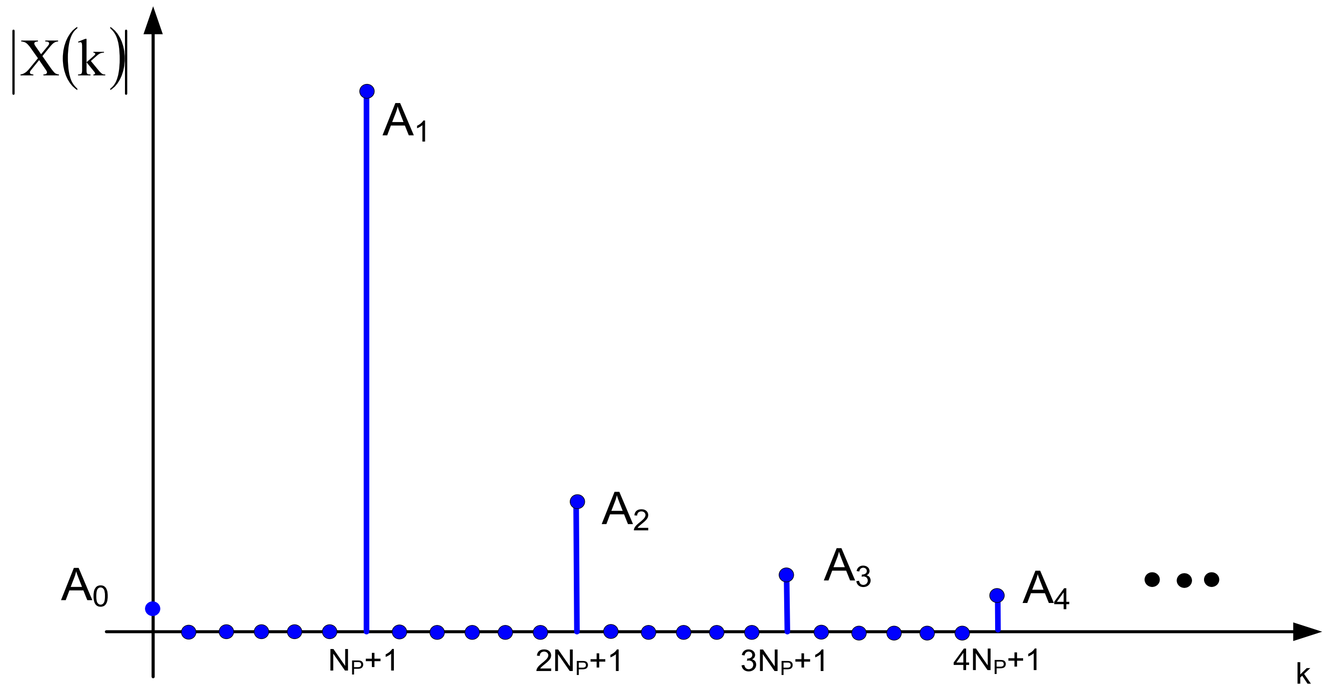
By DFT (with some restrictions that will be discussed)

By FFT (special computational method for obtaining DFT)

Distortion Analysis



If the hypothesis of the theorem are satisfied, we thus have



Review from last lecture

Considerations for Spectral Characterization

- Tool Validation
- DFT Length and NP
- Importance of Satisfying Hypothesis
- Windowing

Considerations for Spectral Characterization

DFT Length and NP

- DFT Length and NP do not affect the computational noise floor
- Although not shown here yet, DFT length does reduce the quantization noise floor coefficients but not total quantization noise

If we assume E_{QUANT} is fixed and no signal present

$$E_{QUANT} \cong \sqrt{\sum_{k=1}^{2^{n_{DFT}}} A_k^2}$$

(these are now the DFT coefficients due to quantization noise, not computation noise)

If the A_k's are constant and equal

$$E_{QUANT} \cong A_k 2^{n_{DFT}/2}$$

Solving for A_k, obtain

$$A_k \cong \frac{E_{QUANT}}{2^{n_{DFT}/2}}$$

If input is full-scale sinusoid with only amplitude quantization with n-bit res,

$$E_{QUANT} \cong \frac{X_{LSB}}{\sqrt{12}} = \frac{X_{REF}}{\sqrt{3} \bullet 2^{n+1}}$$

(this expression is actually independent of input waveform)

Considerations for Spectral Characterization

DFT Length

$$E_{QUANT} \cong \frac{X_{LSB}}{\sqrt{12}} = \frac{X_{REF}}{\sqrt{3} \bullet 2^{n+1}}$$

Substituting for E_{QUANT}, obtain

$$A_k \cong \frac{X_{REF}}{\sqrt{3} \bullet 2^{n+1} 2^{n_{DFT}/2}}$$

This value for A_k thus decreases with the length of the DFT sampline window

Example: if n=16, n_{DFT}=12 (4096 pt transform), and X_{REF}=1V,
then A_k=6.9E-8V (-143dB),

(Note A_k>> computational noise floor (-310dB for Matlab) for all practical n, n_{DFT})

Considerations for Spectral Characterization

- Tool Validation
- DFT Length and NP
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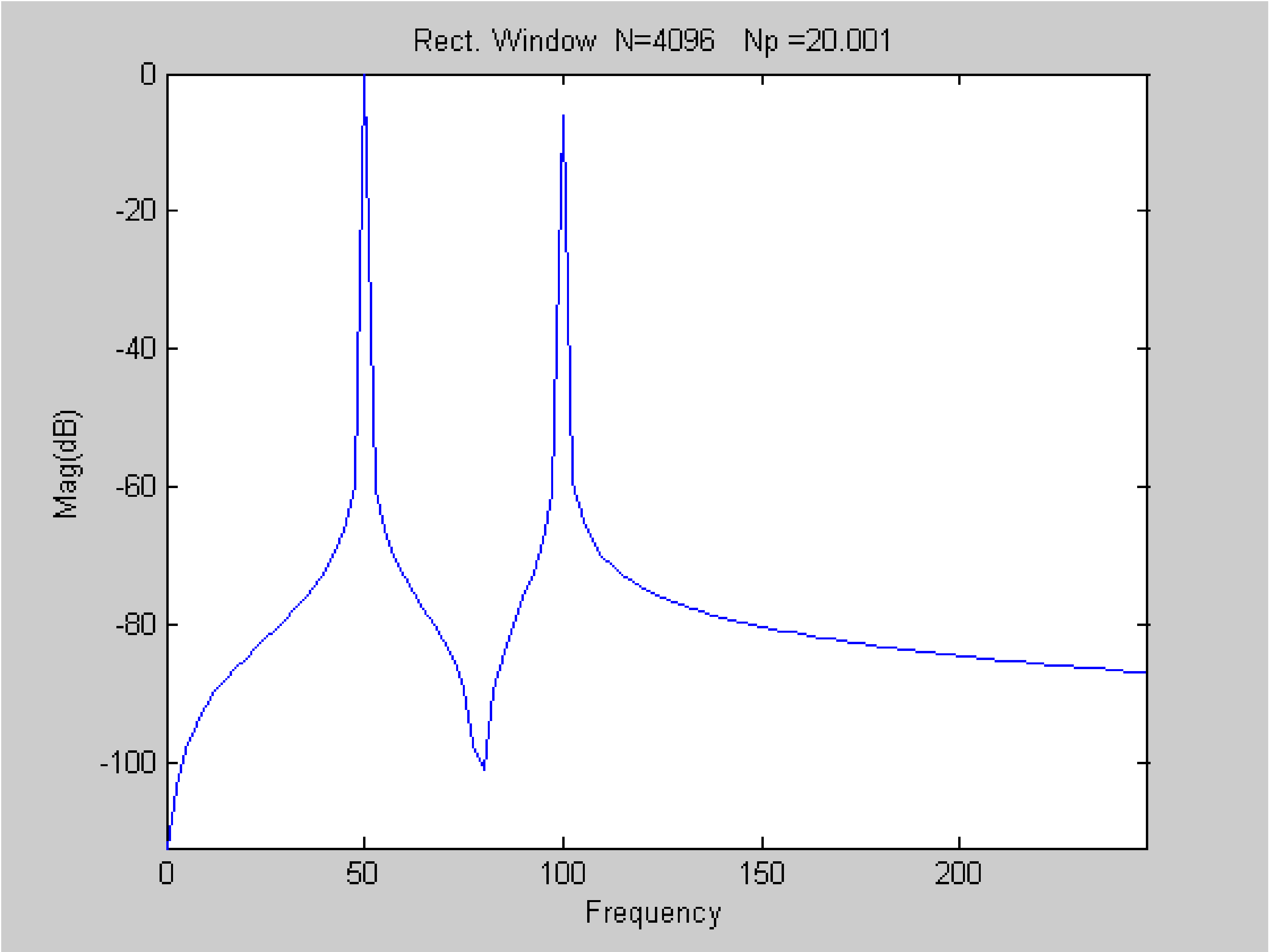
Spectral Response with Non-coherent Sampling



(zoomed in around fundamental)

Considerations for Spectral Characterization

- Tool Validation
- DFT Length and NP
- Importance of Satisfying Hypothesis
  - NP is an integer
  - Band-limited excitation
- Windowing

DFT Examples

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1. The sampling window be an integral number of periods

2. $$N > \frac{2f_{max}}{f_{SIGNAL}} N_P$$

Example $N < \frac{2f_{max}}{f_{SIGNAL}} N_P$ (Not meeting Nyquist sampling rate requirement)

If f_{SIG}=50Hz

and N_P=20 N=512

$$N < \frac{2f_{max}}{f_{SIGNAL}} N_P \quad \Longrightarrow \quad f_{max} < 640\text{Hz}$$

Example

$$N < \frac{2f_{max}}{f_{SIGNAL}} N_P$$

(Not meeting Nyquist sampling rate requirement)

Consider N_P=20 N=512

If f_{SIG}=50Hz but an additional input at 700Hz is present

$$N_P = \frac{NT_S}{T} \quad \leftrightarrow \quad f_{SAMP} = f_{SIGNAL}\frac{N}{N_P} \qquad f_{SAMP} = 1.280KHz$$

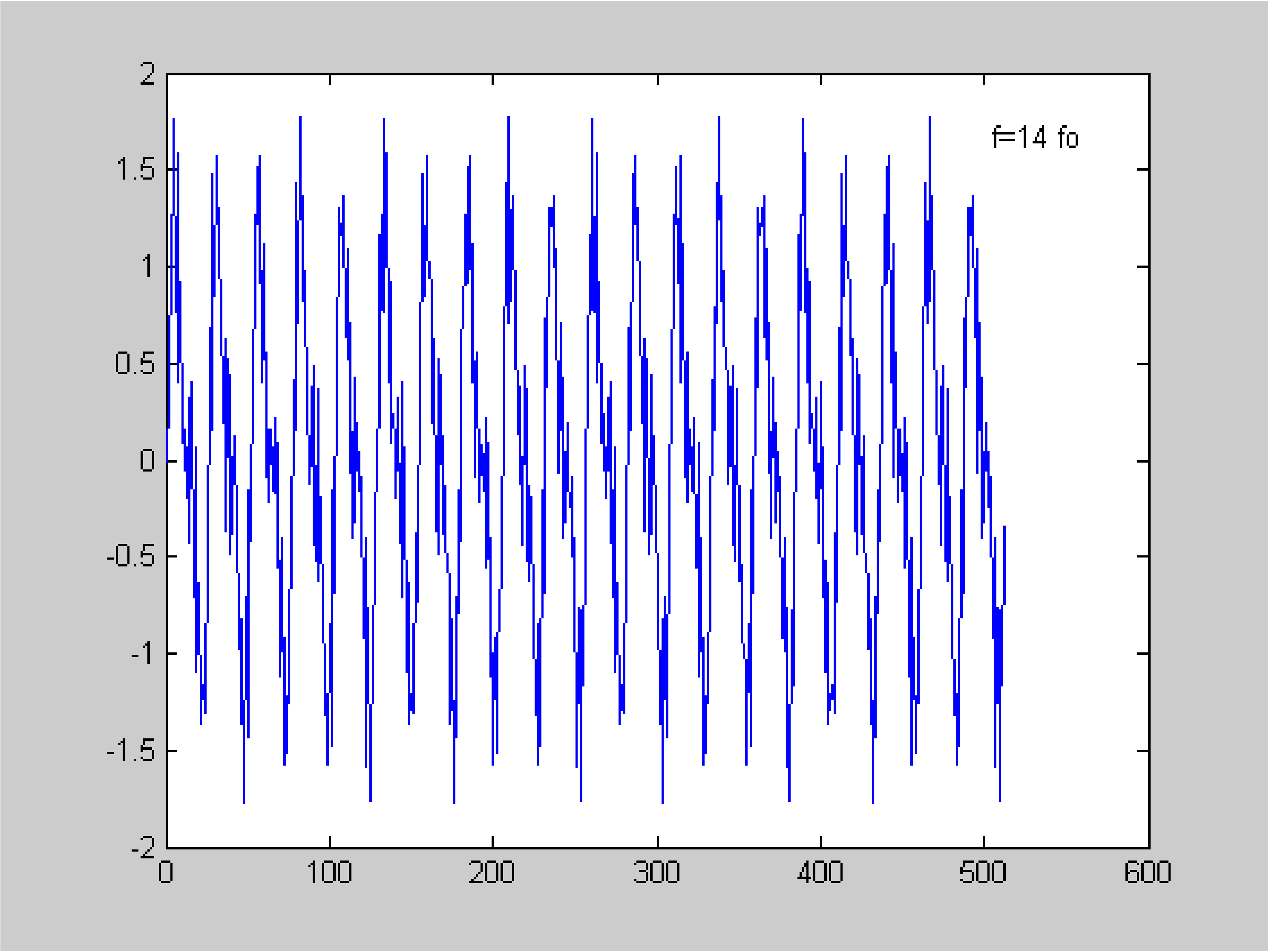
$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t) + 0.5\sin(14\omega t)$$

$$\omega = 2\pi f_{SIG}$$

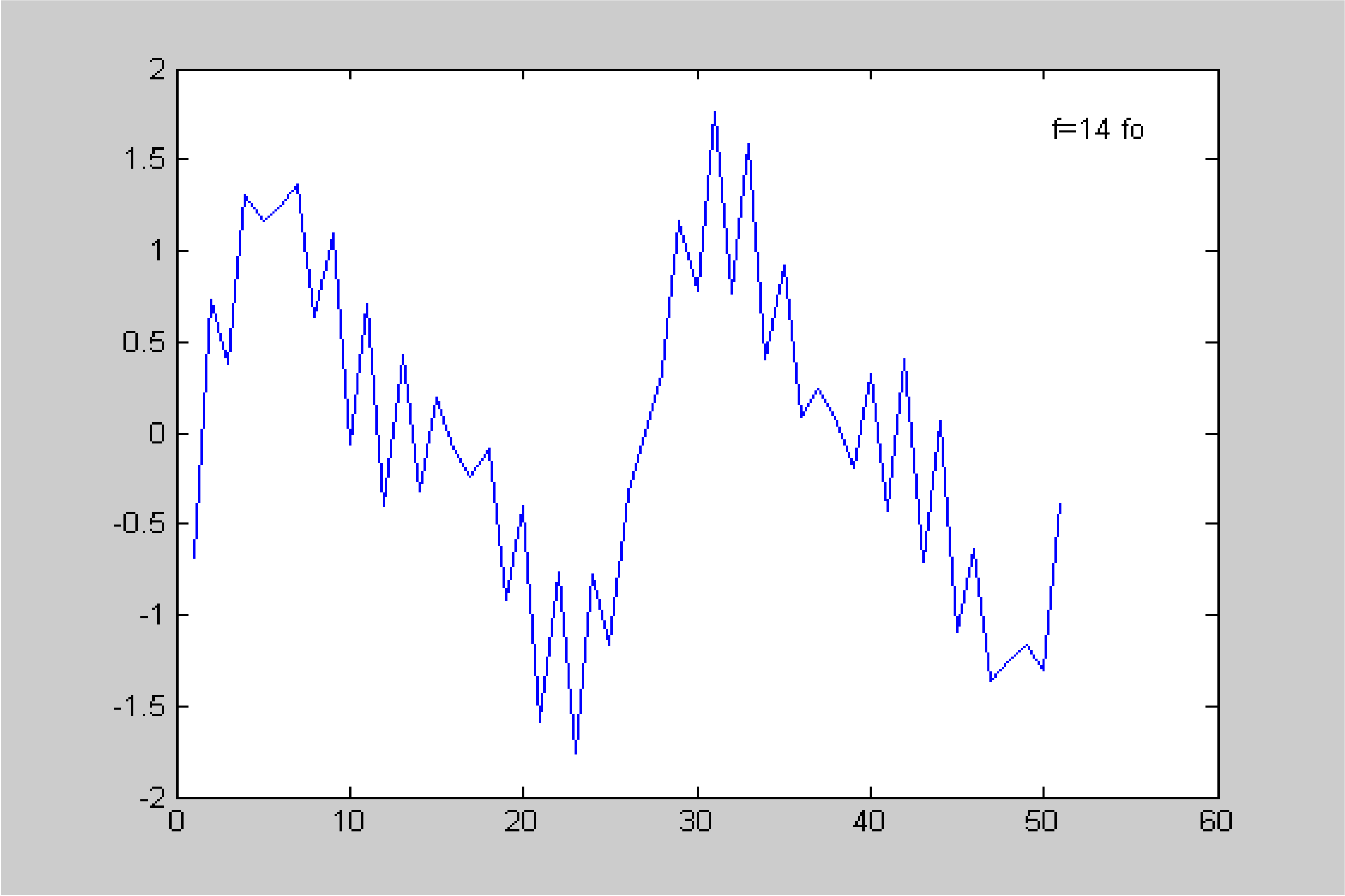
(i.e. the component at 700 Hz which violates the band limit requirement – Nyquist rate for the 700 Hz input is 1.4KHz)

Recall $20\log_{10}(0.5)=-6.0205999$

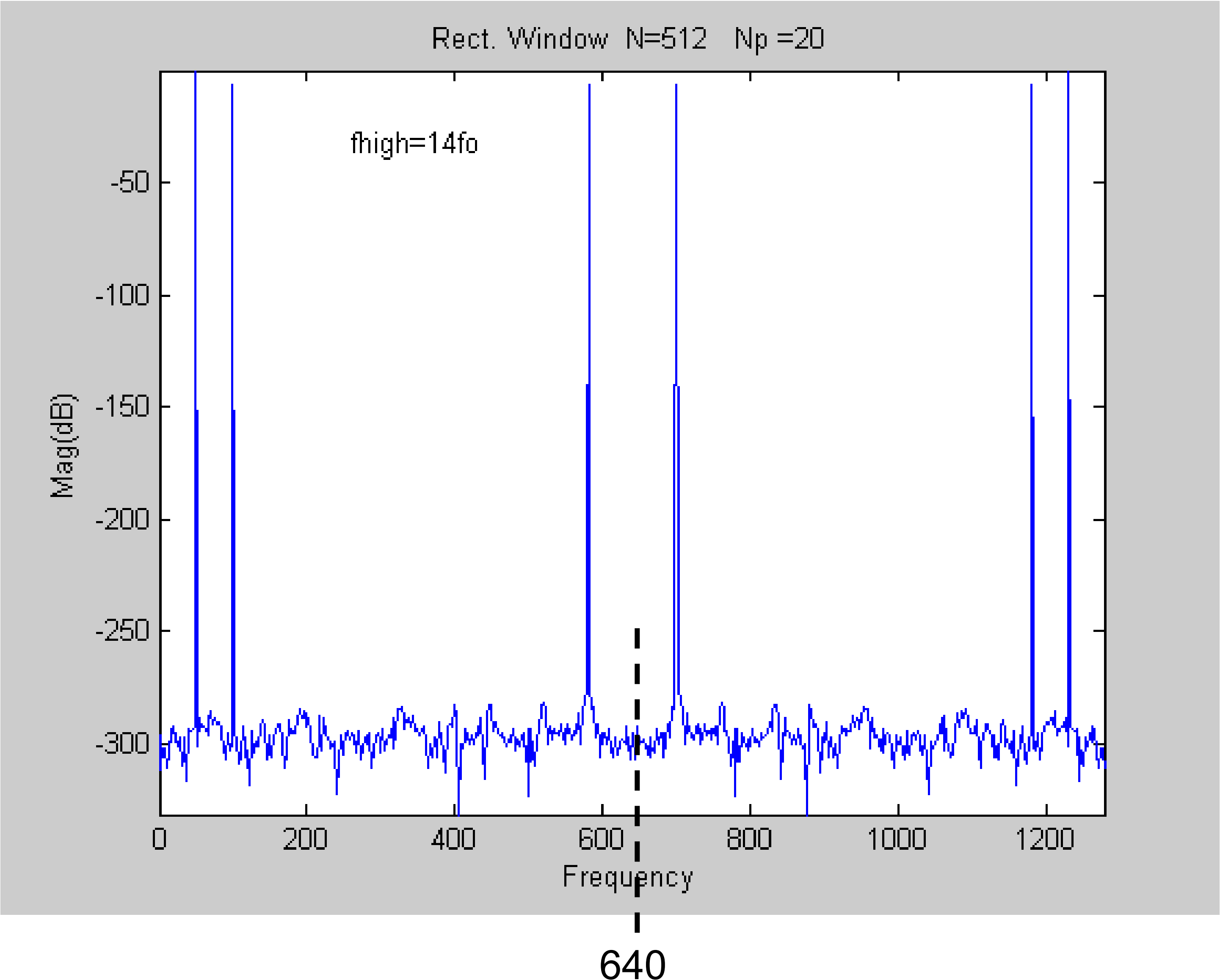
Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components

$f_{high}=14fo$

Columns 1 through 7

-296.9507 -311.9710 -302.4715 -302.1545 -310.8392 -304.5465 -293.9310

Columns 8 through 14

-299.0778 -292.3045 -297.0529 -301.4639 -297.3332 -309.6947 -308.2308

Columns 15 through 21

-297.3710 -316.5113 -293.5661 -294.4045 -293.6881 -292.6872 $\boxed{-0.0000}$

Columns 22 through 28

-301.6889 -288.4812 -292.5621 -292.5853 -294.1383 -296.4034 -289.5216

Columns 29 through 35

-285.9204 -292.1676 -289.0633 -292.1318 -290.6342 -293.2538 -296.8434

Effects of High-Frequency Spectral Components

f_{high}=14fo

Columns 36 through 42

-301.7087 -307.2119 -295.1726 -303.4403 -301.6427 $\boxed{-6.0206}$ -295.3018

Columns 43 through 49

-298.9215 -309.4829 -306.7363 -293.0808 -300.0882 -306.5530 -302.9962

Columns 50 through 56

-318.4706 -294.8956 -304.4663 -300.8919 -298.7732 -301.2474 -293.3188

Effects of High-Frequency Spectral Components

Aliased components at $$f_{alias} = f_{sample} - f$$

$$f_{alias} = 25.6 f_{sig} - 14 f_{sig} = 11.6 f_{sig}$$

$$thus\ \ position\ in\ sequence = 1 + N_p \frac{f_{alias}}{f_{sig}} = 1 + 20 \bullet 11.6 = 233$$

Columns 225 through 231

-296.8883 -292.8175 -295.8882 -286.7494 -300.3477 -284.4253 -282.7639

Columns 232 through 238

-273.9840 -6.0206 -274.2295 -284.4608 -283.5228 -297.6724 -291.7545

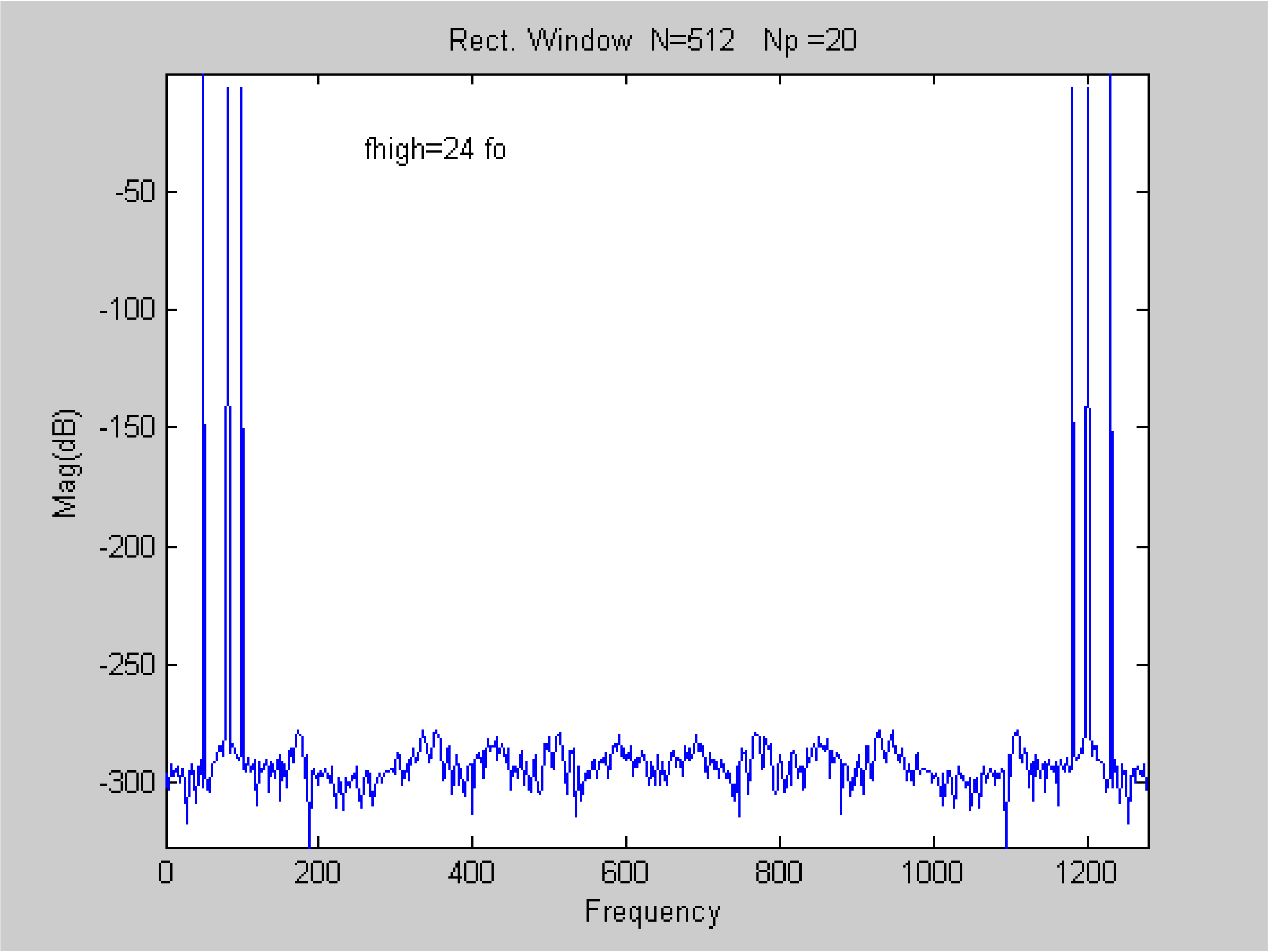
Columns 239 through 245

-299.1299 -305.8361 -295.1772 -295.1670 -300.2698 -293.6406 -304.2886

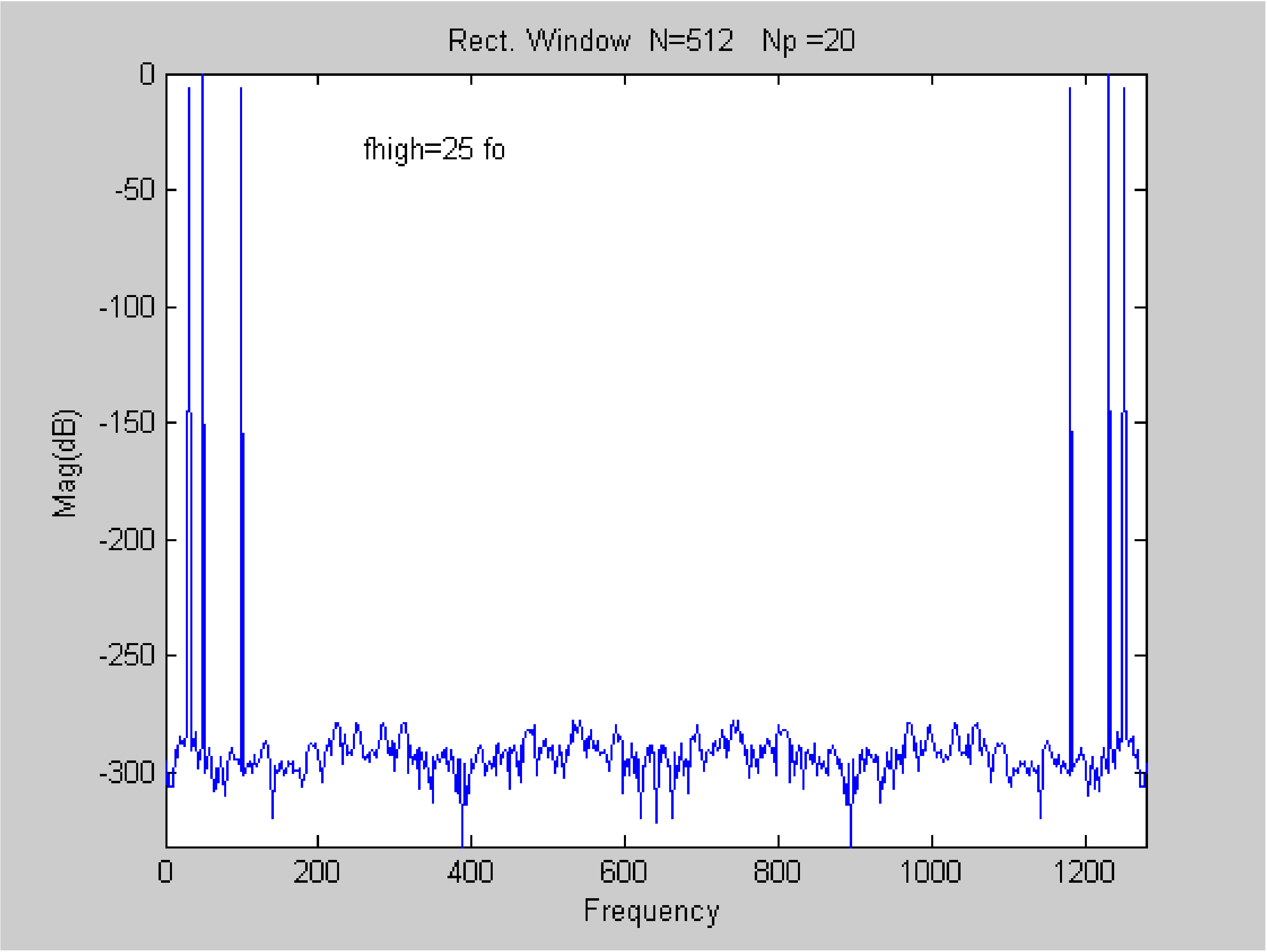
Columns 246 through 252

-302.0233 -306.6100 -297.7242 -305.4513 -300.4242 -298.1795 -299.0956

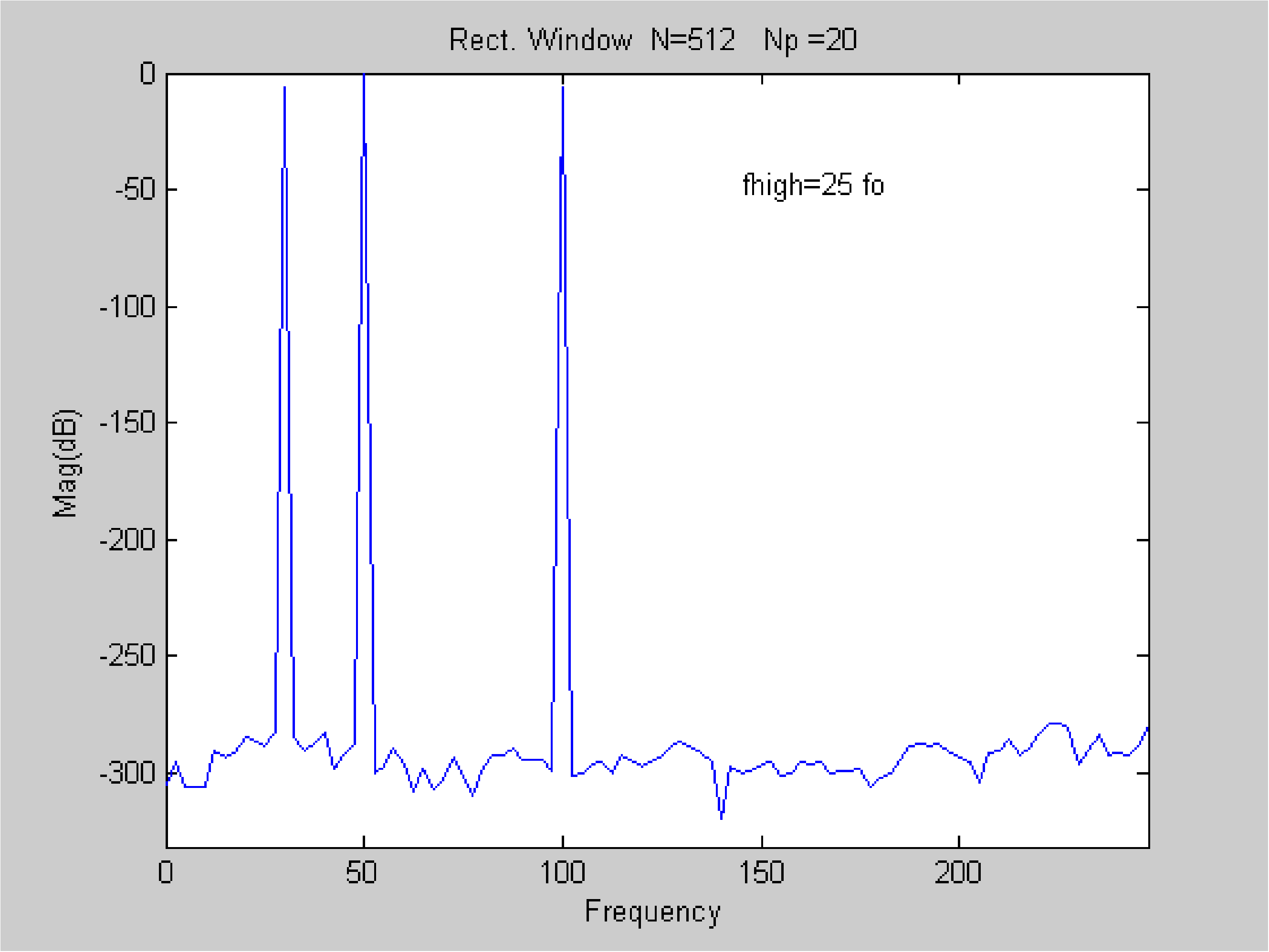
Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components

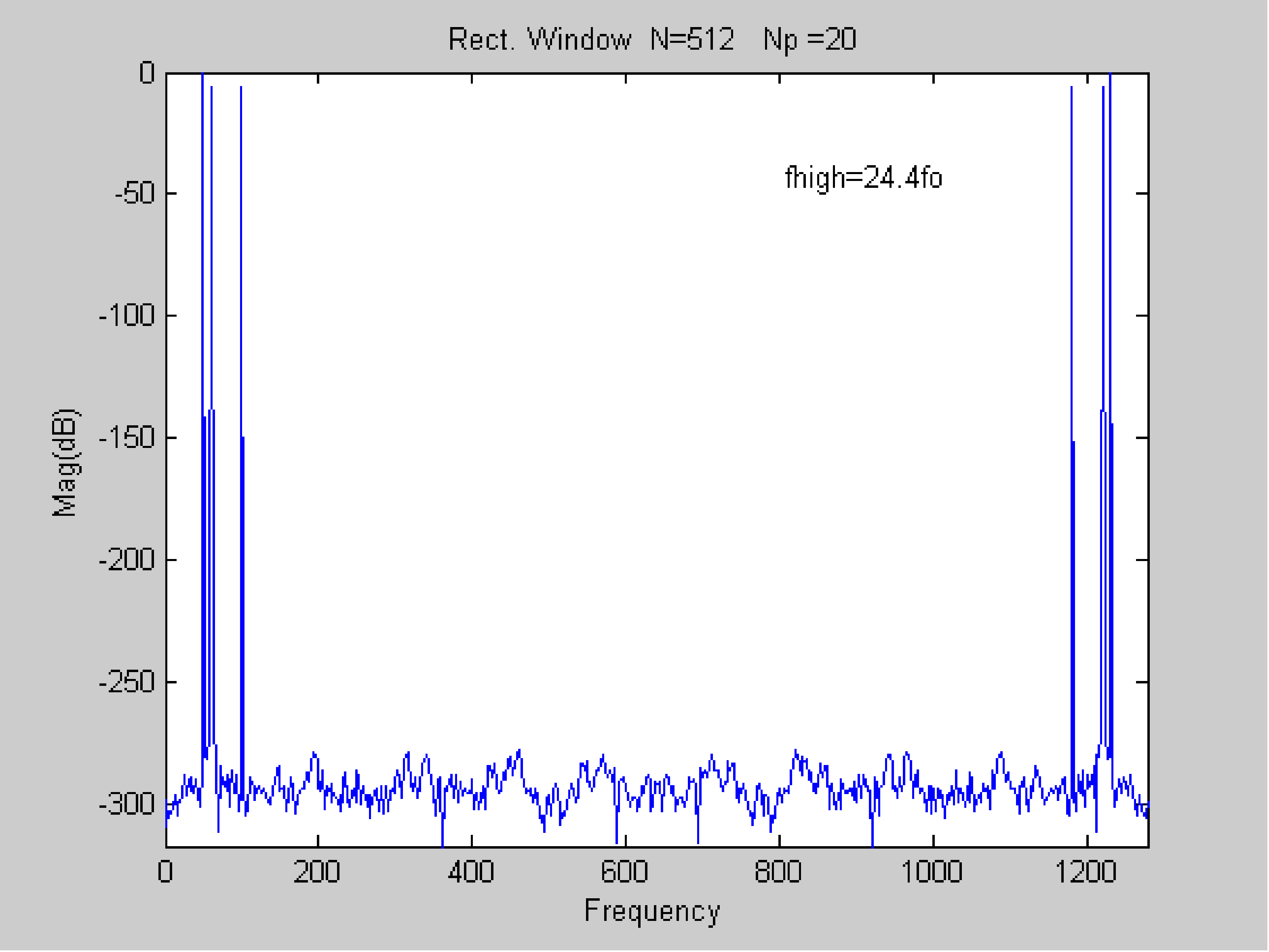


Effects of High-Frequency Spectral Components

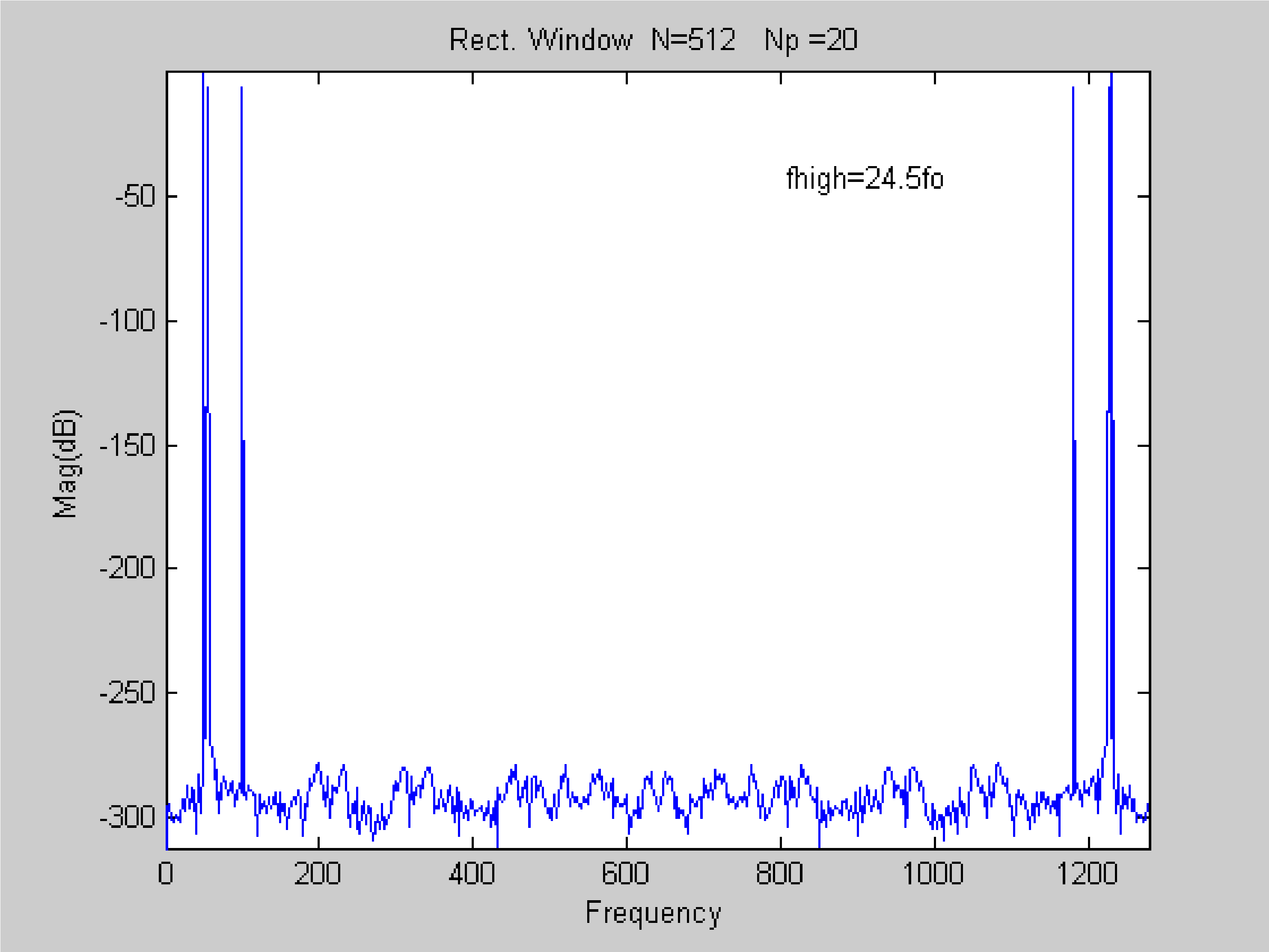


(zoomed in around fundamental)

Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components



Observations

- Aliasing will occur if the band-limited part of the hypothesis for using the DFT is not satisfied
- Modest aliasing will cause high frequency components that may or may not appear at a harmonic frequency
- More egregious aliasing can introduce components near or on top of fundamental and lower-order harmonics
- Important to avoid aliasing if the DFT is used for spectral characterization

Review Questions

Q1: How many DFT terms are there if the sample window is of length 4096?

A: 4096

Q2: When the magnitude of the DFT coefficients are plotted, the horizontal axis is an index axis (i.e. dimensionless) but often the index terms are labeled as frequency terms. If the sampling frequency is f_s and N samples are taken, what is the frequency of the first DFT term? What is the frequency of the 2^{nd} DFT term?

A: 0 Hz A: fs/N

Q3: If samples of the time-domain signal are made over exactly 31 periods, which index term corresponds to the fundamental? To the second harmonic?

A: 32^{nd} term A: 63^{rd} term

Q4: What is the difference between the DFT and the FFT**?**

A: FFT is a computationally efficient method of computing the DFT

Q5: True or False: The DFT terms are real numbers.

A: False We are, however, often interested most in the magnitude of the DFT terms and these are real

Q6: True or False: The magnitude of the DFT terms are symmetric around index number N/2. A: Yes

Considerations for Spectral Characterization

- Tool Validation
- DFT Length and NP
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Considerations for Spectral Characterization

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Are there any strategies to address the problem of requiring precisely an integral number of periods to use the FFT?

Windowing is sometimes used

Windowing is sometimes misused

Windowing

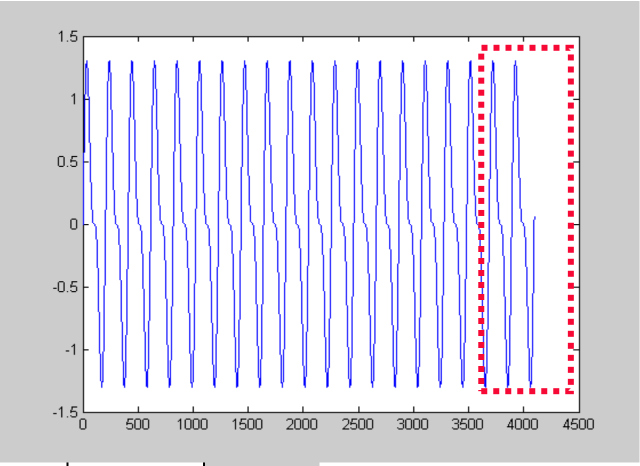
Windowing is the weighting of the time domain function to maintain continuity at the end points of the sample window

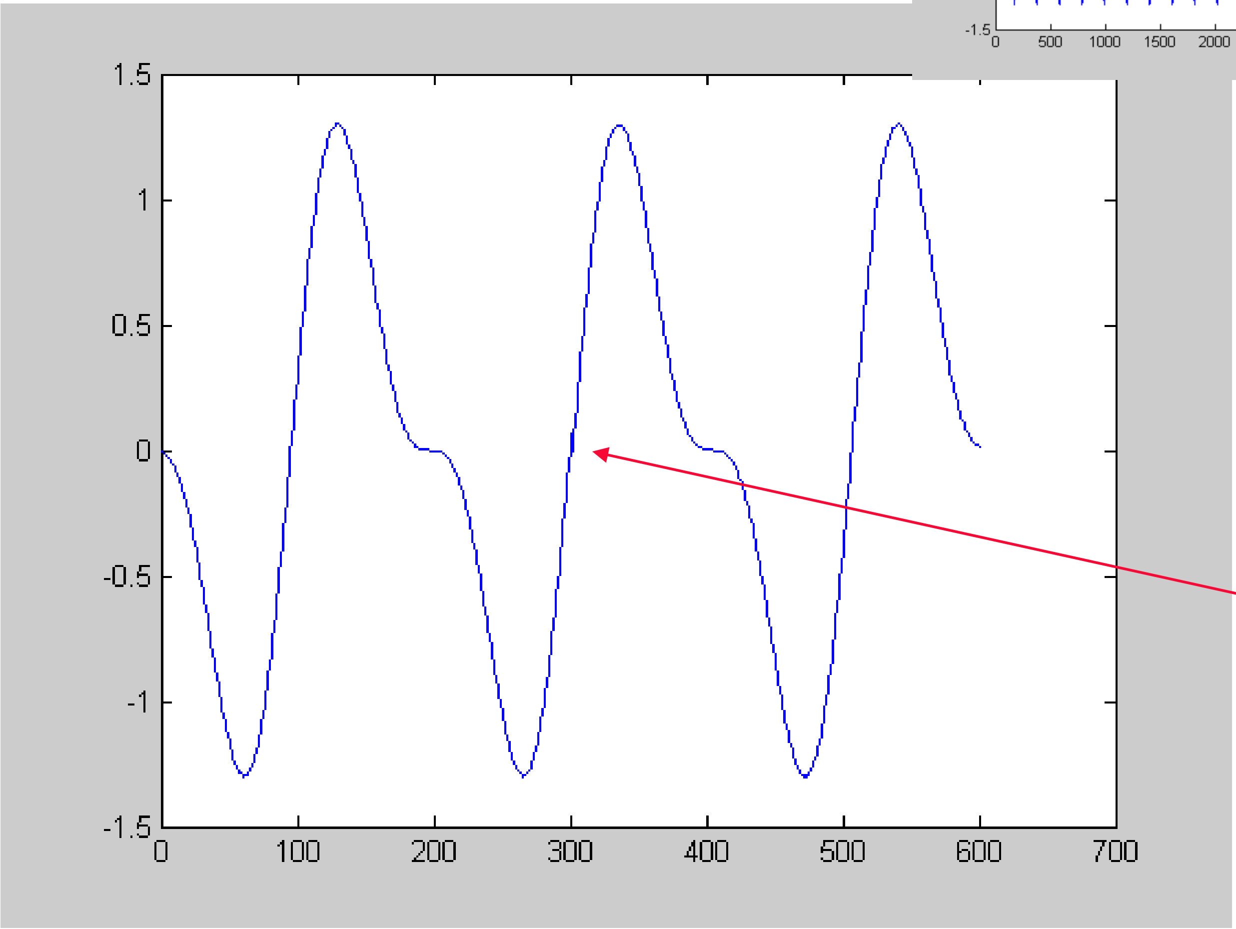
Well-studied window functions:

- Rectangular (also with appended zeros)
- Triangular
- Hamming
- Hanning
- Blackman

Recall

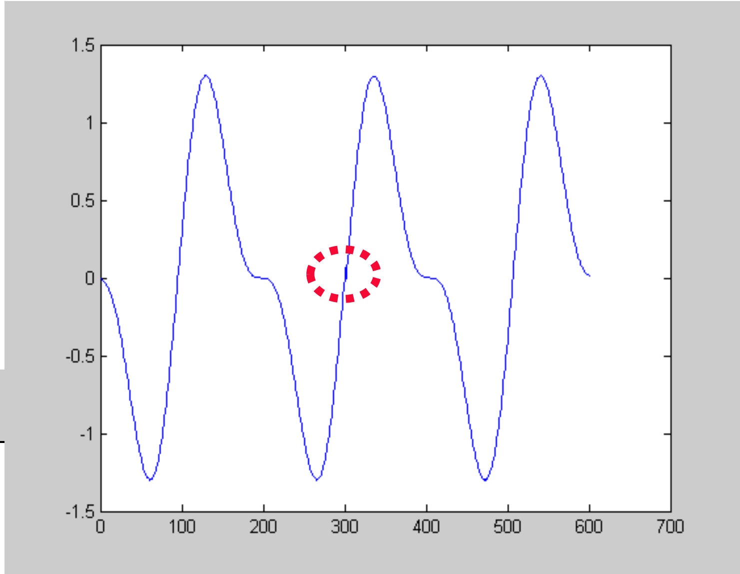
Input Waveform

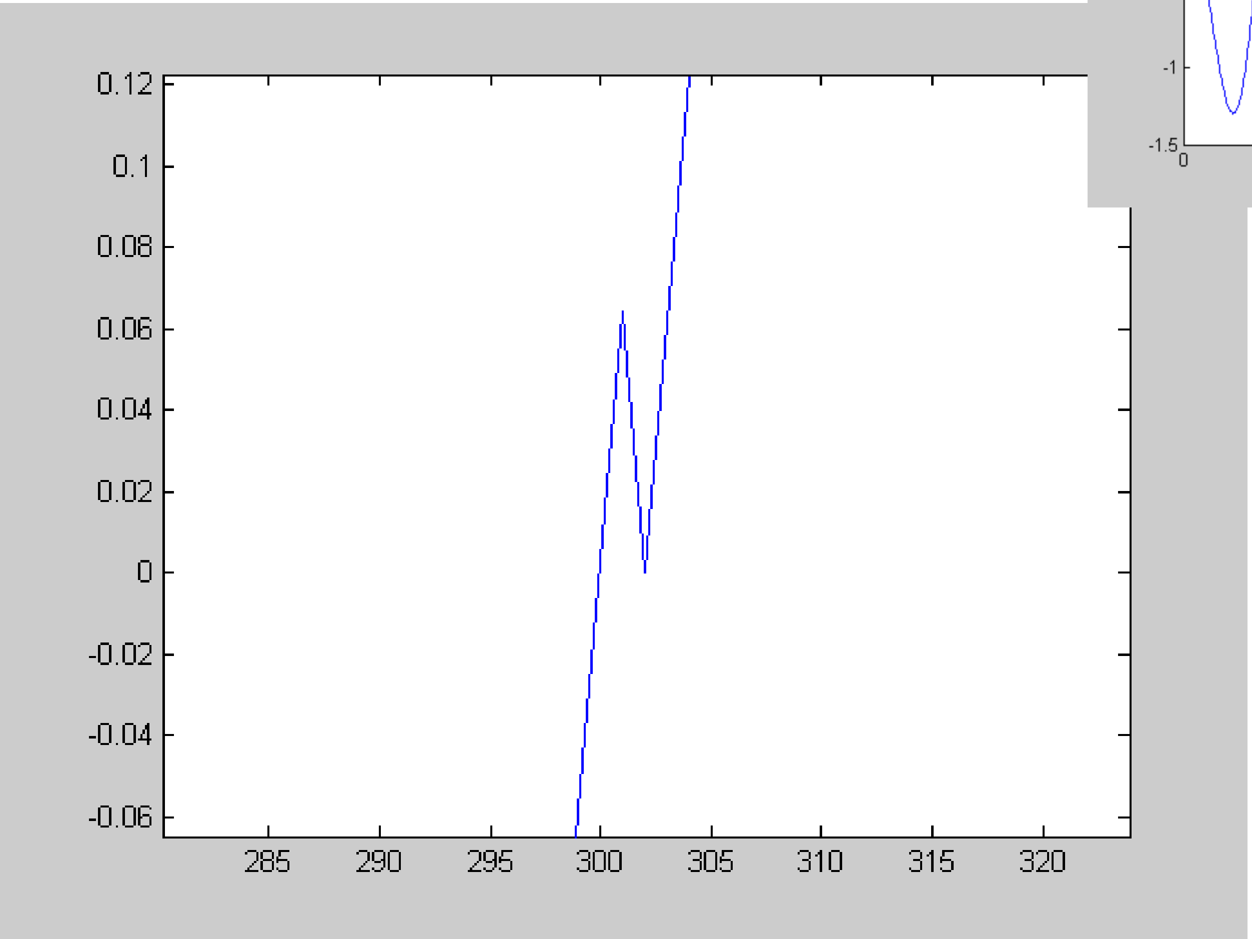




Recall

Input Waveform





Rectangular Window

Sometimes termed a boxcar window

Uniform weight

Can append zeros

Without appending zeros equivalent to no window

Rectangular Window

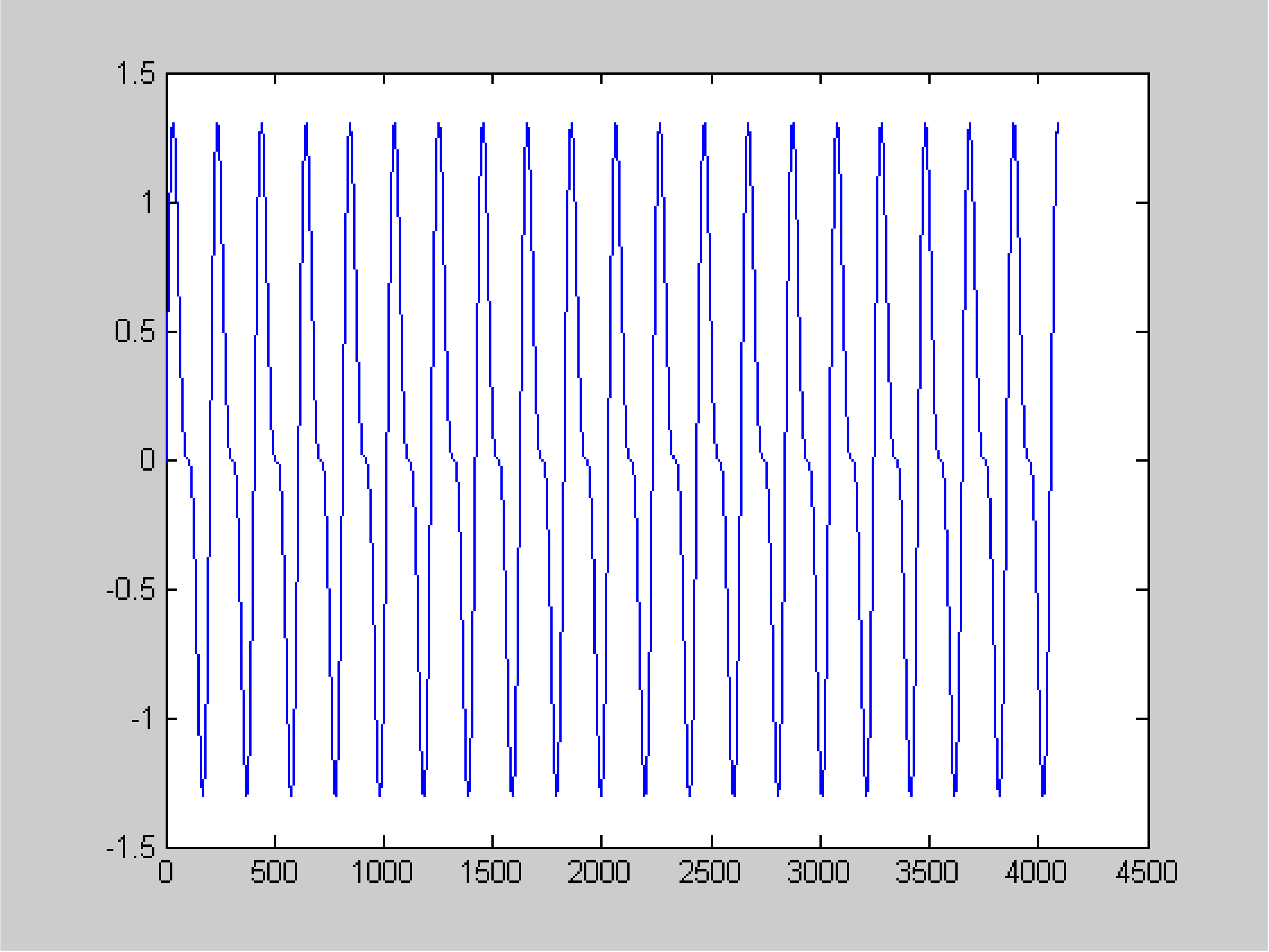
Assume f_{SIG}=50Hz

$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

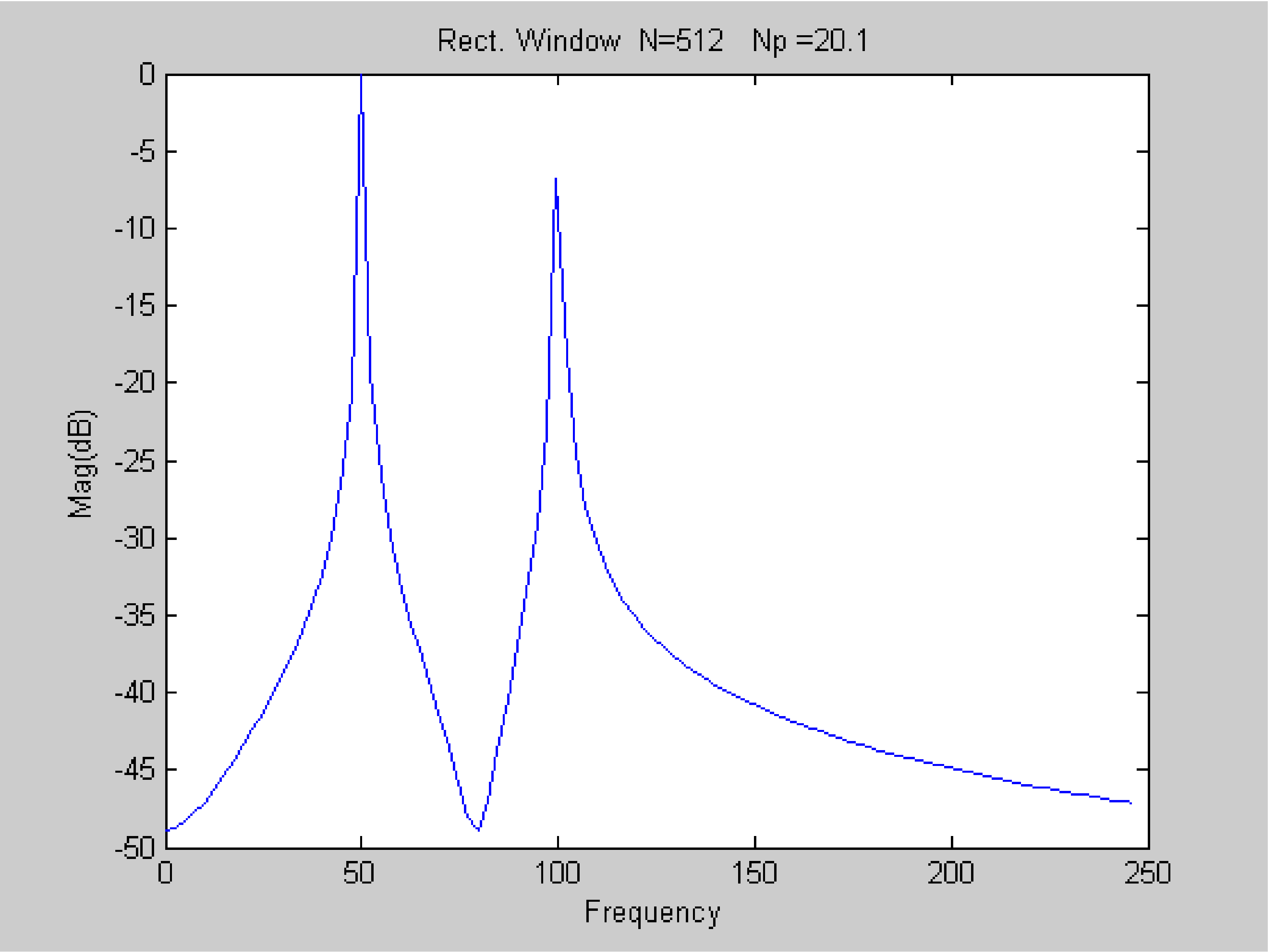
$$\omega = 2\pi f_{SIG}$$

Consider N_P=20.1 N=512

Rectangular Window

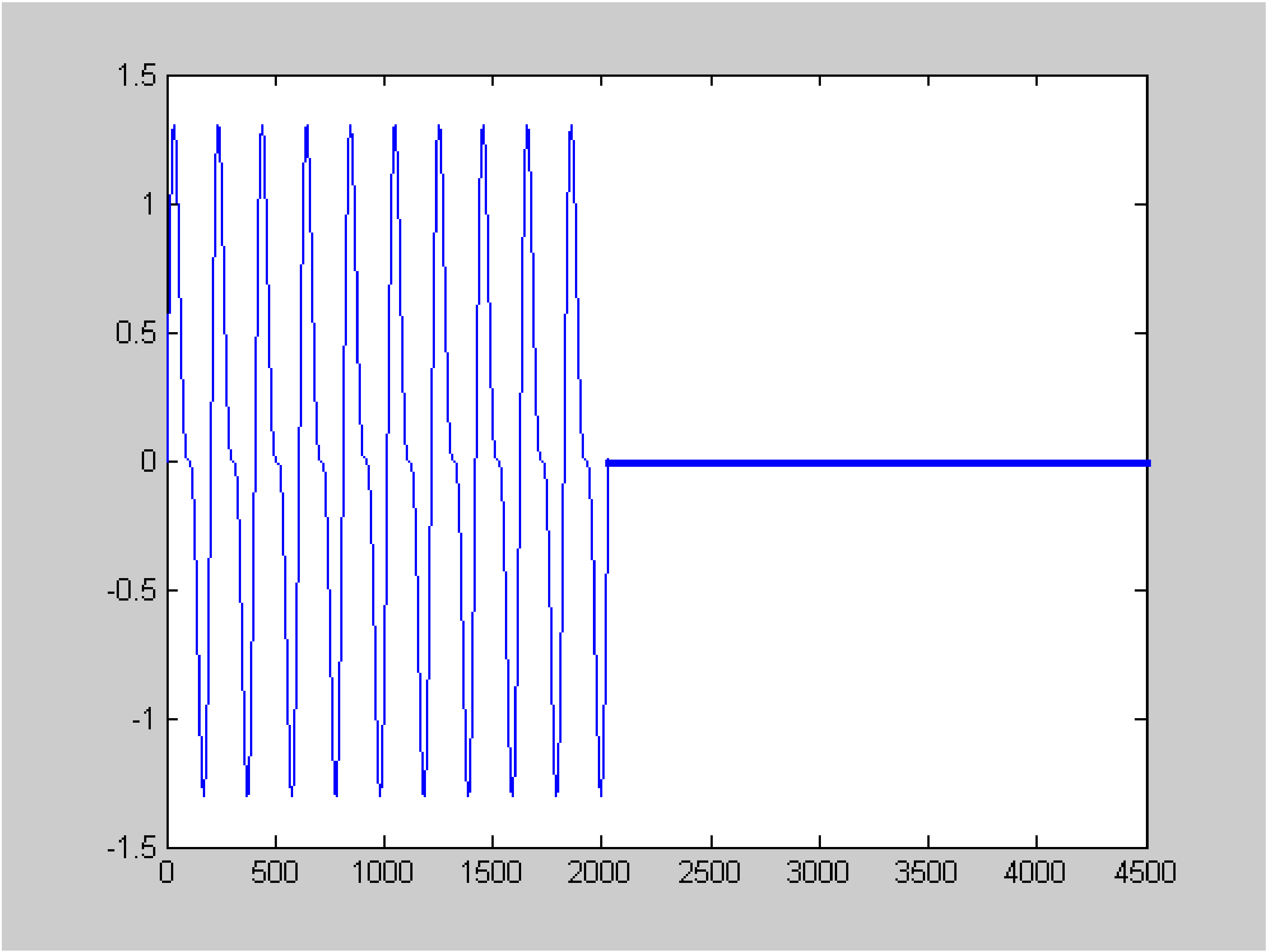


Spectral Response with Non-coherent sampling



(zoomed in around fundamental)

Rectangular Window (with appended zeros)



Rectangular Window

Columns 1 through 7

-48.8444	-48.7188	-48.3569	-47.7963	-47.0835	-46.2613	-45.3620

Columns 8 through 14

-44.4065	-43.4052	-42.3602	-41.2670	-40.1146	-38.8851	-37.5520

Columns 15 through 21

-36.0756	-34.3940	-32.4043	-29.9158	-26.5087	-20.9064	-0.1352

Columns 22 through 28

-19.3242	-25.9731	-29.8688	-32.7423	-35.1205	-37.2500	-39.2831

Columns 29 through 35

-41.3375	-43.5152	-45.8626	-48.0945	-48.8606	-46.9417	-43.7344

Rectangular Window

Columns 1 through 7

-48.8444	-48.7188	-48.3569	-47.7963	-47.0835	-46.2613	-45.3620

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Columns 15 through 21

-36.0756	-34.3940	-32.4043	-29.9158	-26.5087	-20.9064	-0.1352

Columns 22 through 28

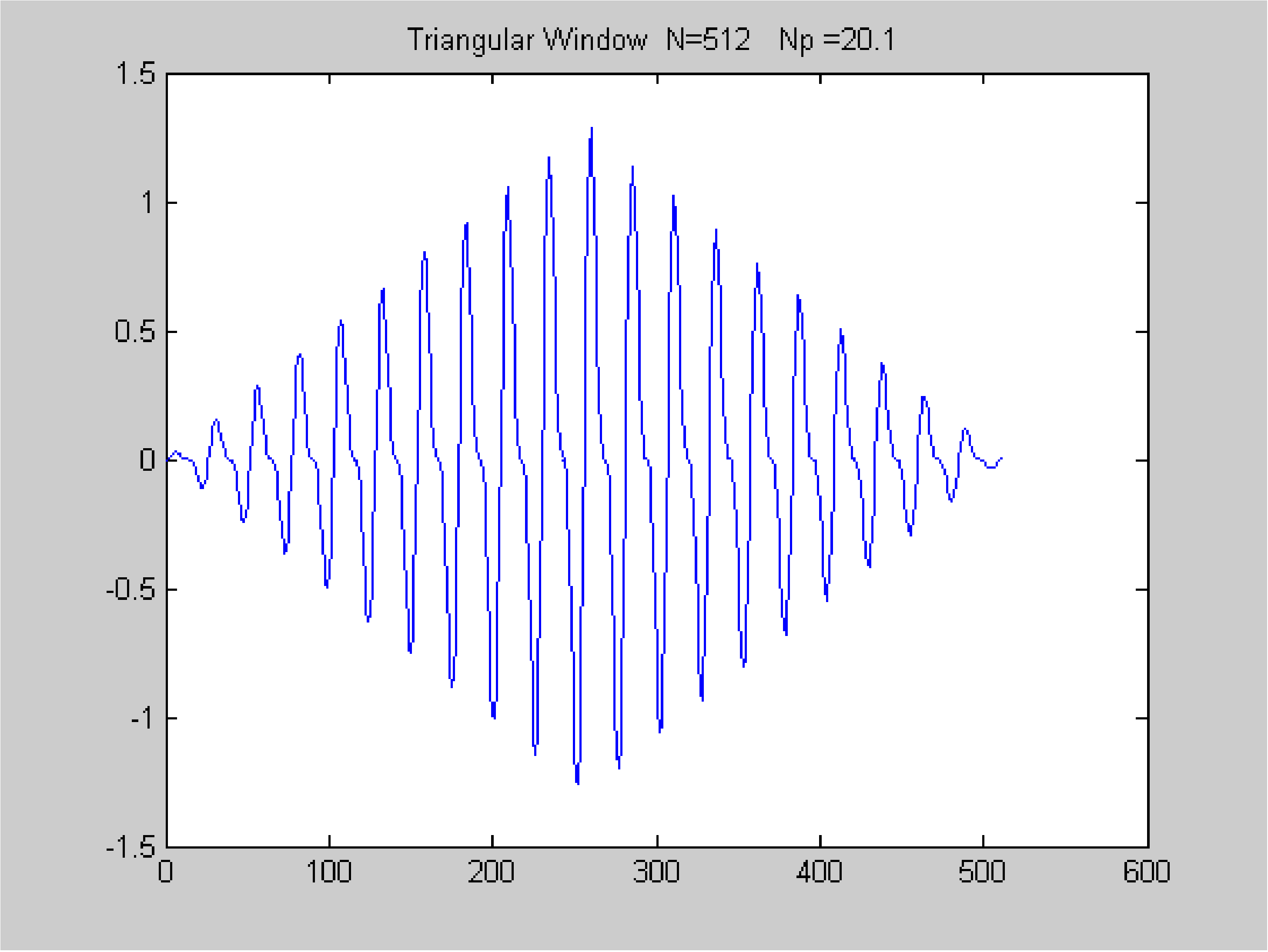
-19.3242	-25.9731	-29.8688	-32.7423	-35.1205	-37.2500	-39.2831

Columns 29 through 35

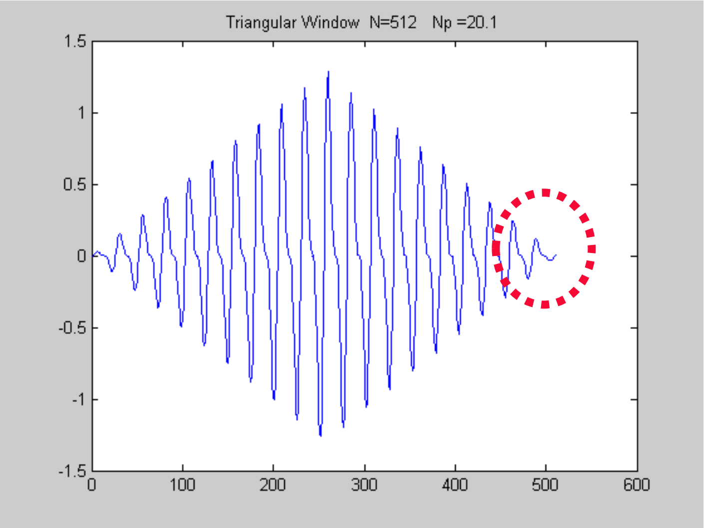
-41.3375	-43.5152	-45.8626	-48.0945	-48.8606	-46.9417	-43.7344

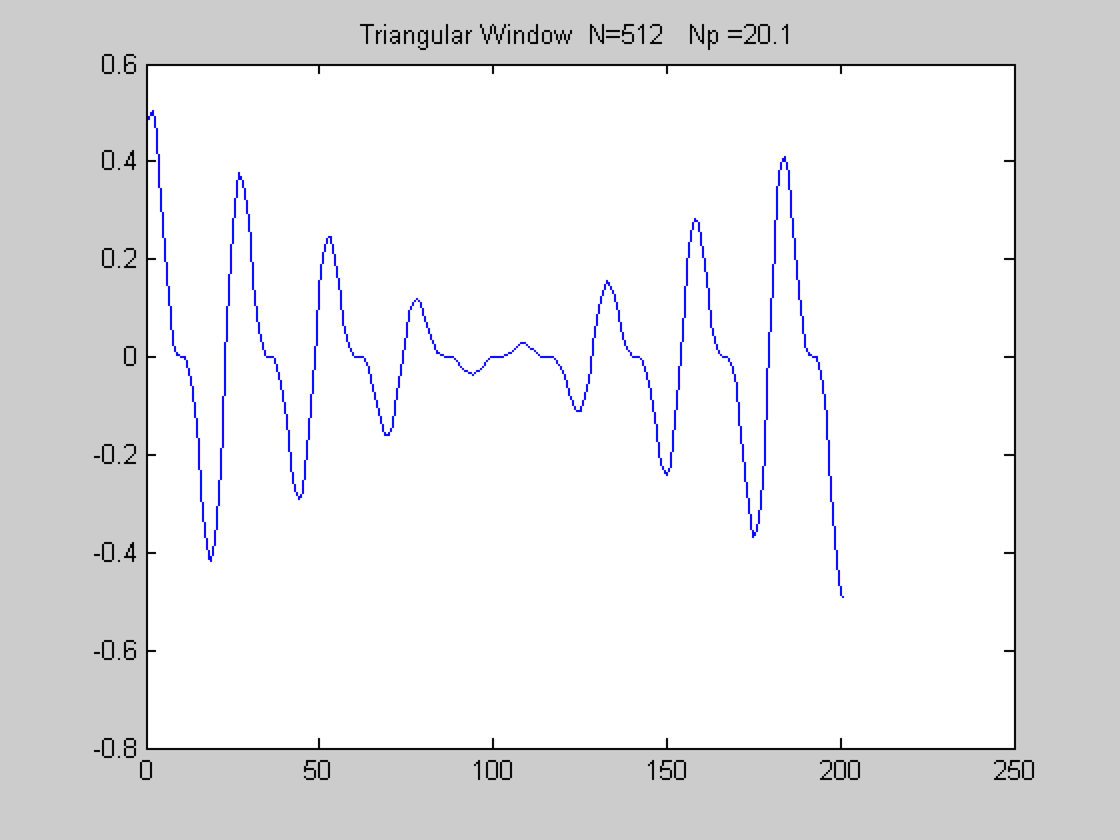
Energy spread over several frequency components

Triangular Window

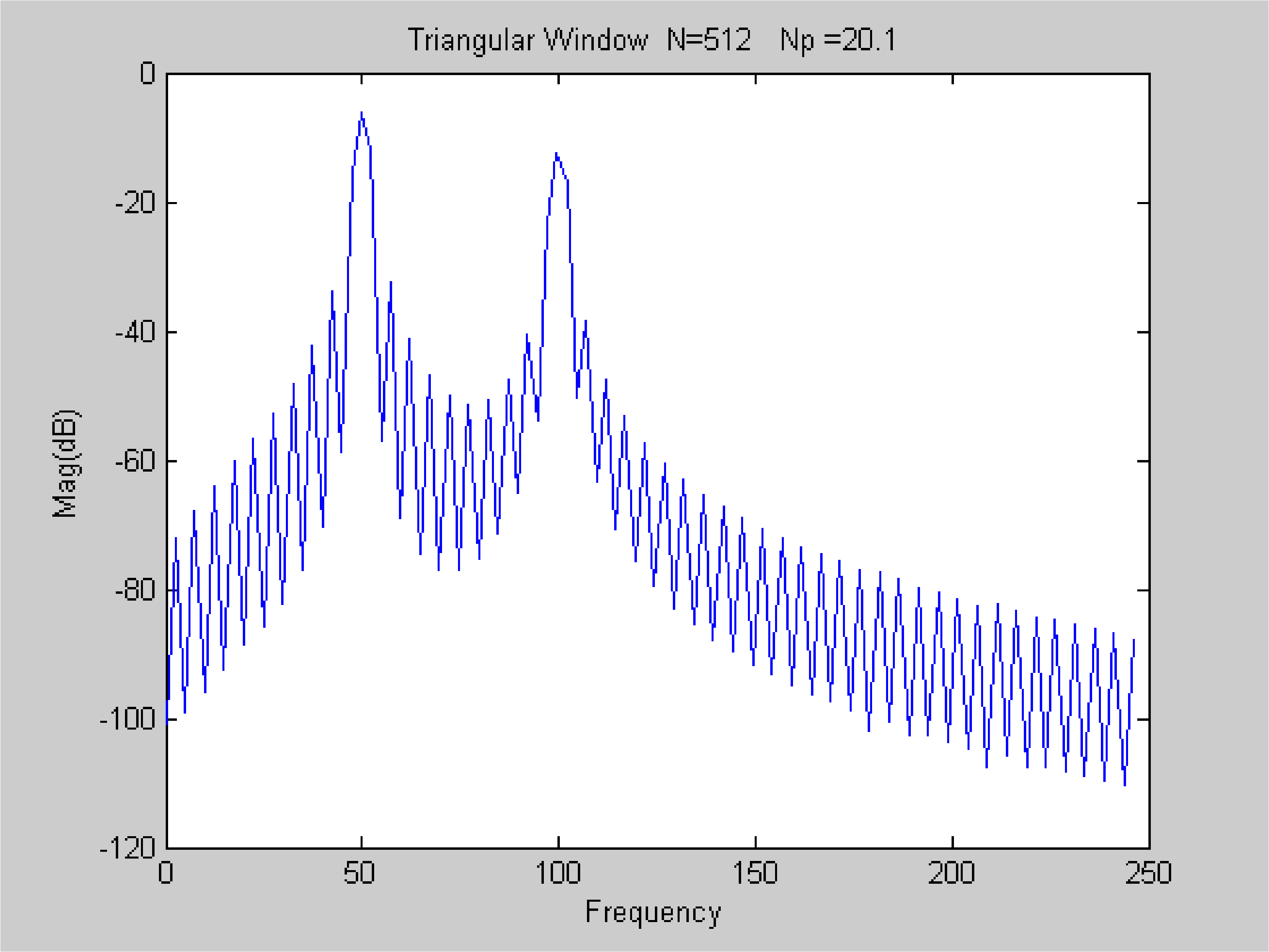


Triangular Window



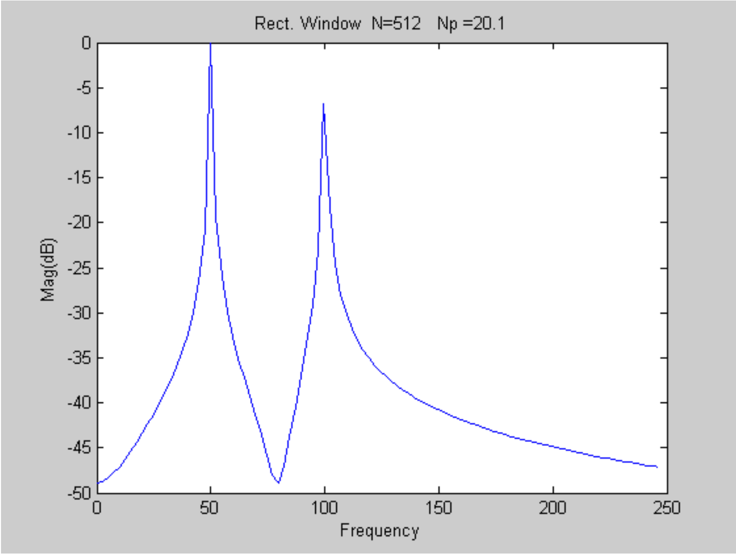


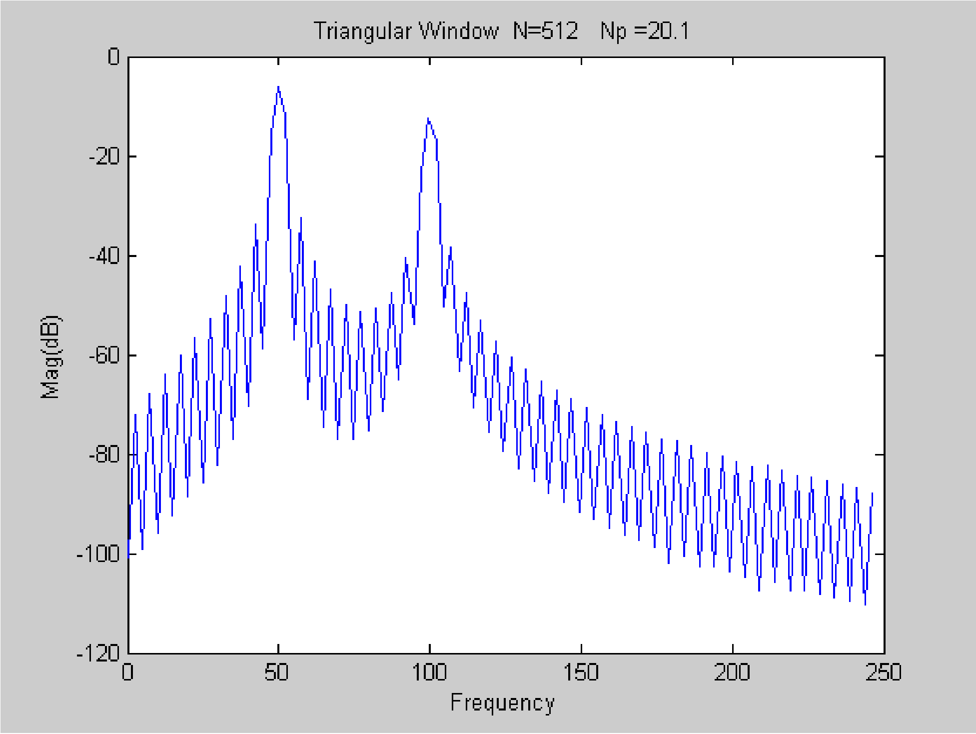
Spectral Response with Non-Coherent Sampling and Windowing



(zoomed in around fundamental)

Triangular Window





Triangular Window

Columns 1 through 7

-100.8530 -72.0528 -99.1401 -68.0110 -95.8741 -63.9944 -92.5170

Columns 8 through 14

-60.3216 -88.7000 -56.7717 -85.8679 -52.8256 -82.1689 -48.3134

Columns 15 through 21

-77.0594 -42.4247 -70.3128 -33.7318 -58.8762 -15.7333 -6.0918

Colu

-12.

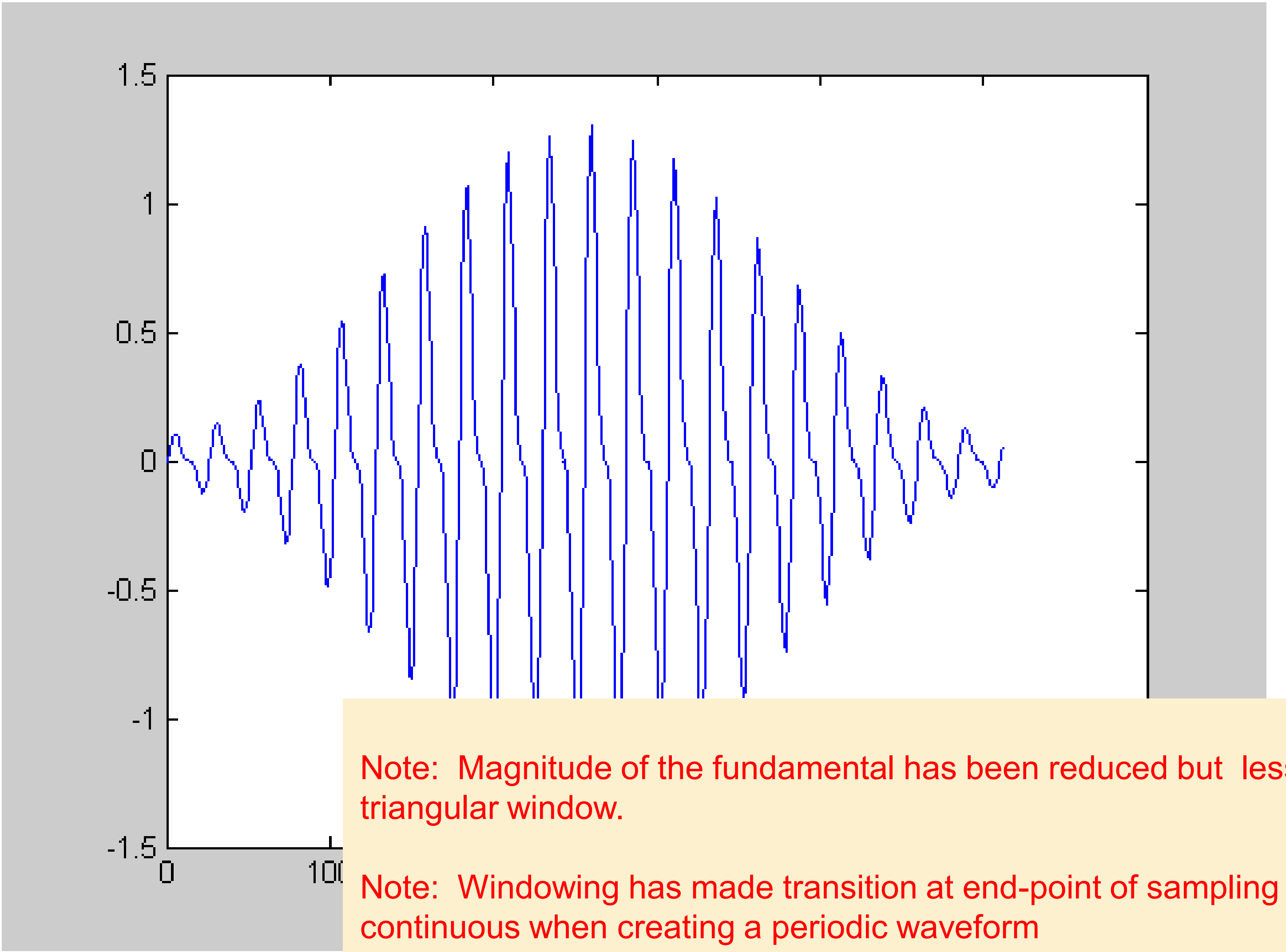
Colu

-77.

Note: Magnitude of the fundamental has been reduced but so have the skirting effects have also been reduced.

Note: Windowing has reduced energy in the signal but also made transition at end-point of sampling window continuous when creating a periodic waveform

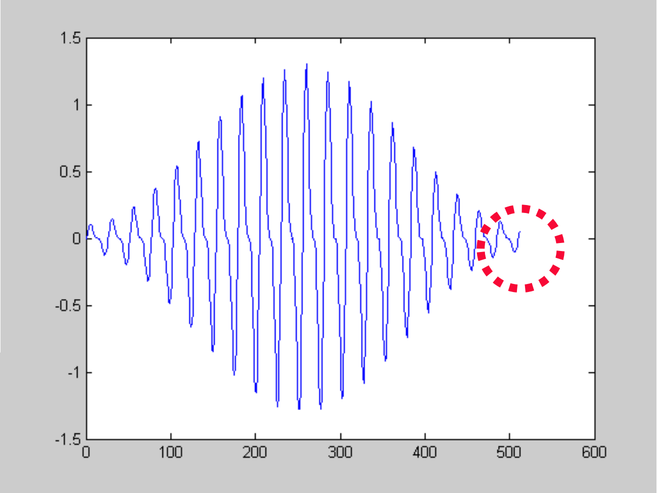
Hamming Window

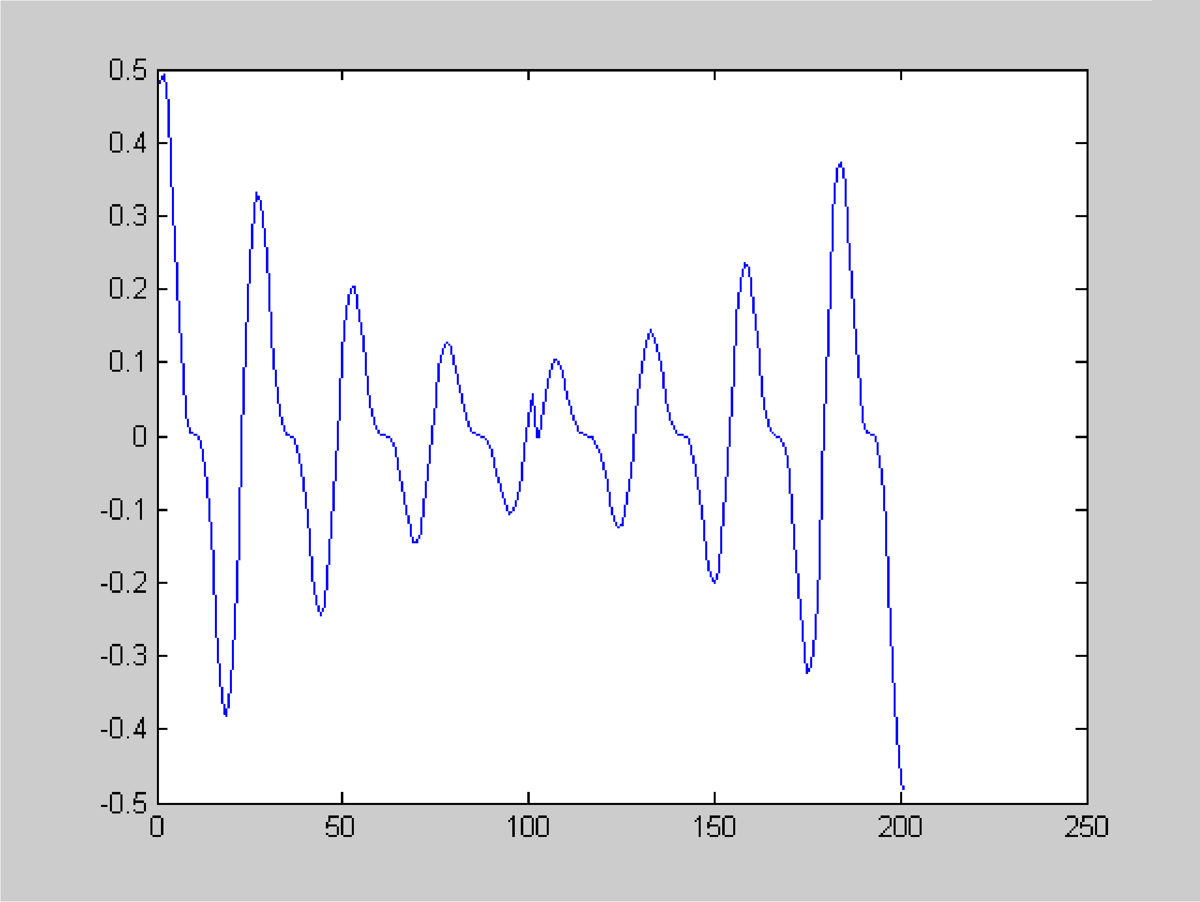


Note: Magnitude of the fundamental has been reduced but less than for triangular window.

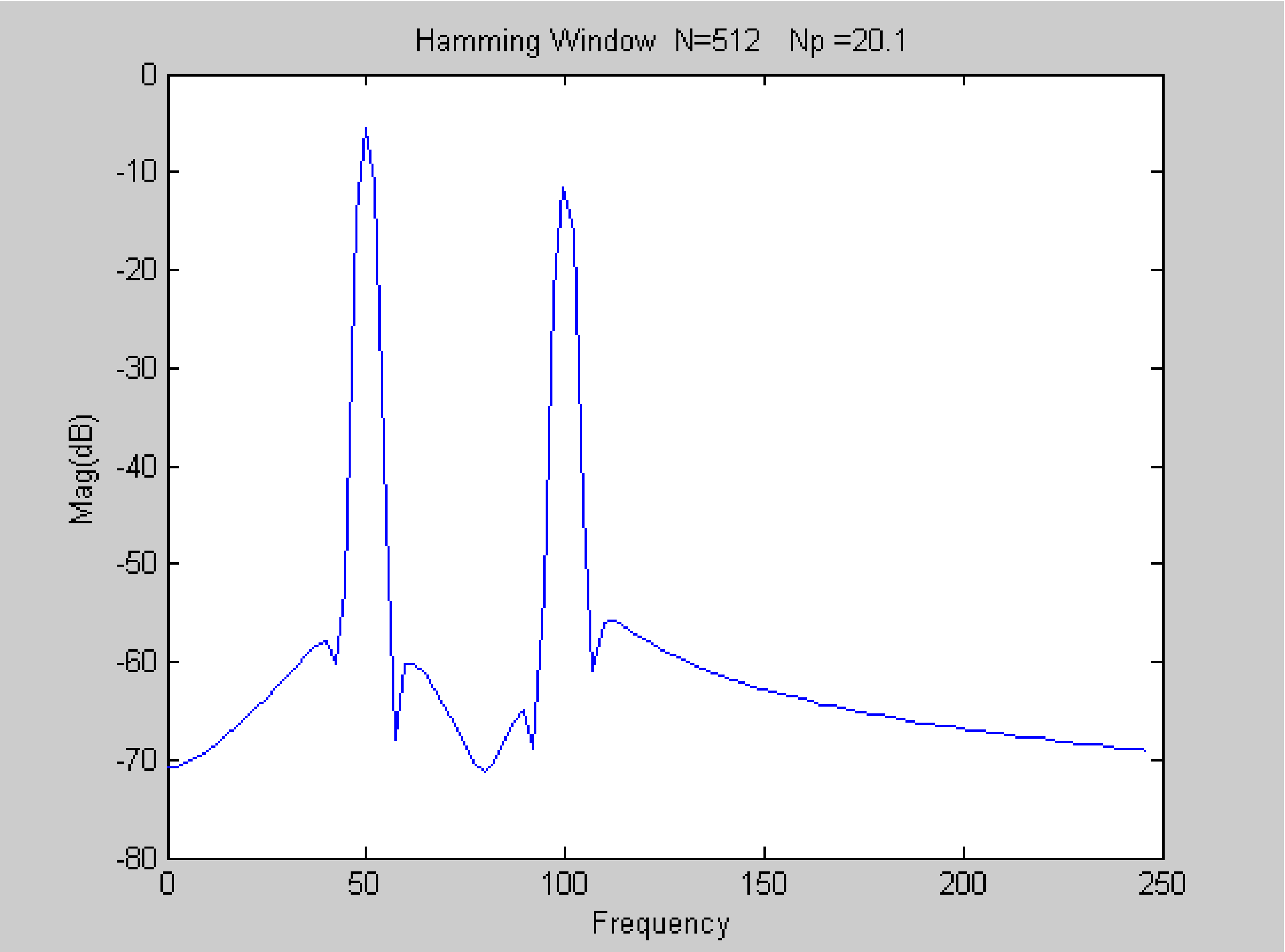
Note: Windowing has made transition at end-point of sampling window continuous when creating a periodic waveform

Hamming Window



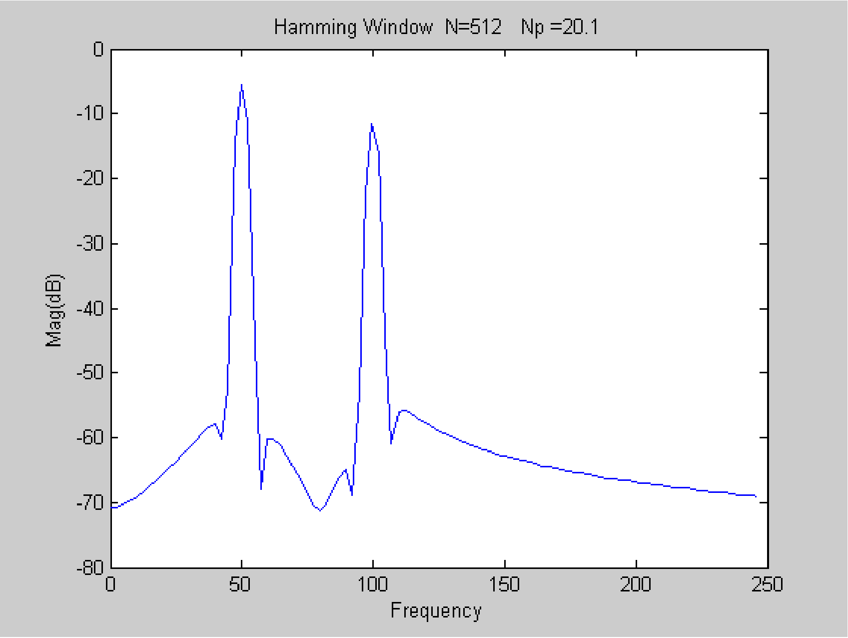


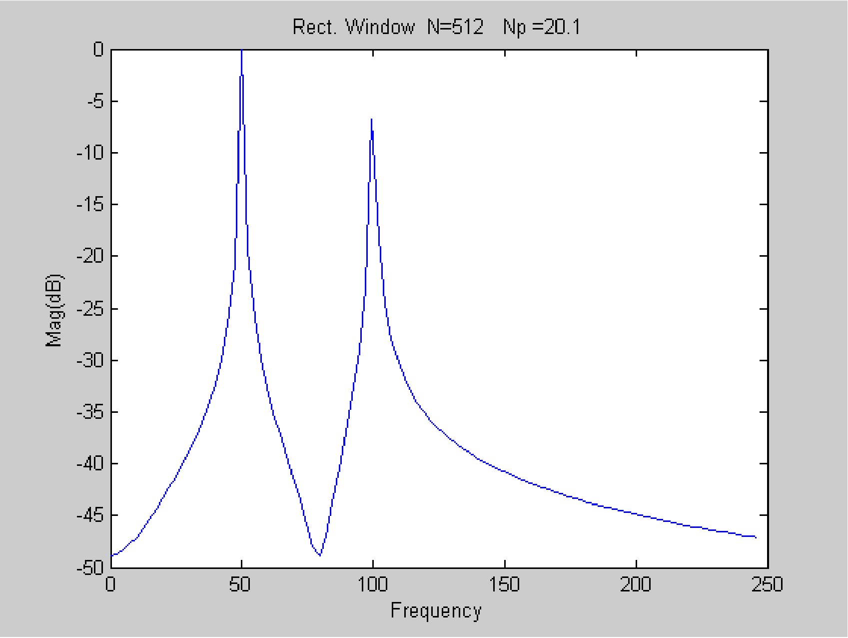
Spectral Response with Non-Coherent Sampling and Windowing



(zoomed in around fundamental)

Comparison with Rectangular Window





Note: Vertical axis are different

Hamming Window

Columns 1 through 7

-70.8278 -70.6955 -70.3703 -69.8555 -69.1502 -68.3632 -67.5133

Columns 8 through 14

-66.5945 -65.6321 -64.6276 -63.6635 -62.6204 -61.5590 -60.4199

Columns 15 through 21

-59.3204 -58.3582 -57.8735 -60.2994 -52.6273 -14.4702 -5.4343

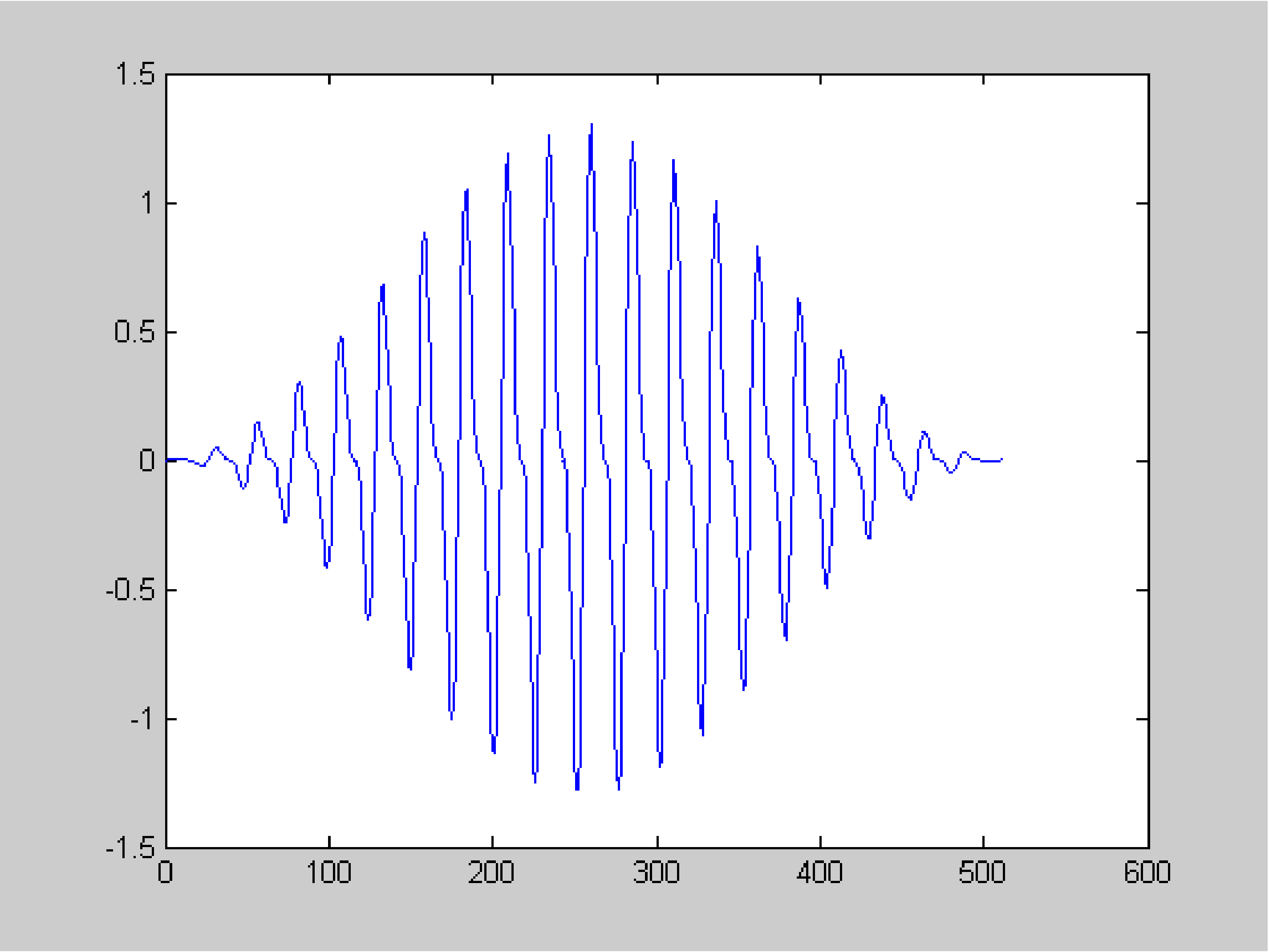
Columns 22 through 28

-11.2659 -45.2190 -67.9926 -60.1662 -60.1710 -61.2796 -62.7277

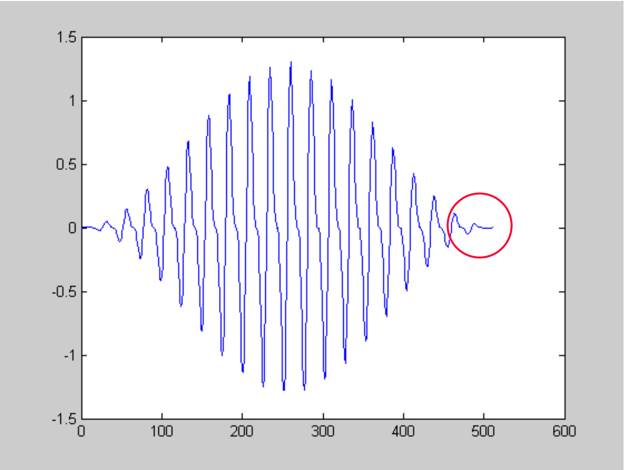
Columns 29 through 35

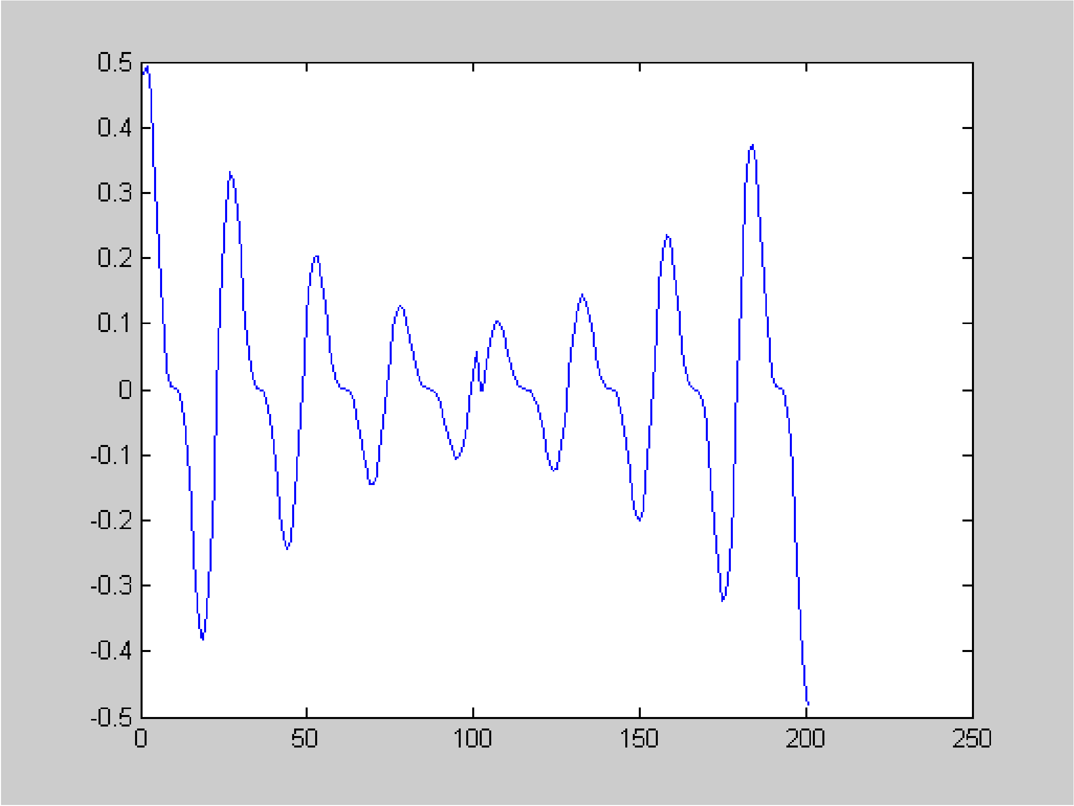
-64.3642 -66.2048 -68.2460 -70.1835 -71.1529 -70.2800 -68.1145

Hanning Window



Hanning Window





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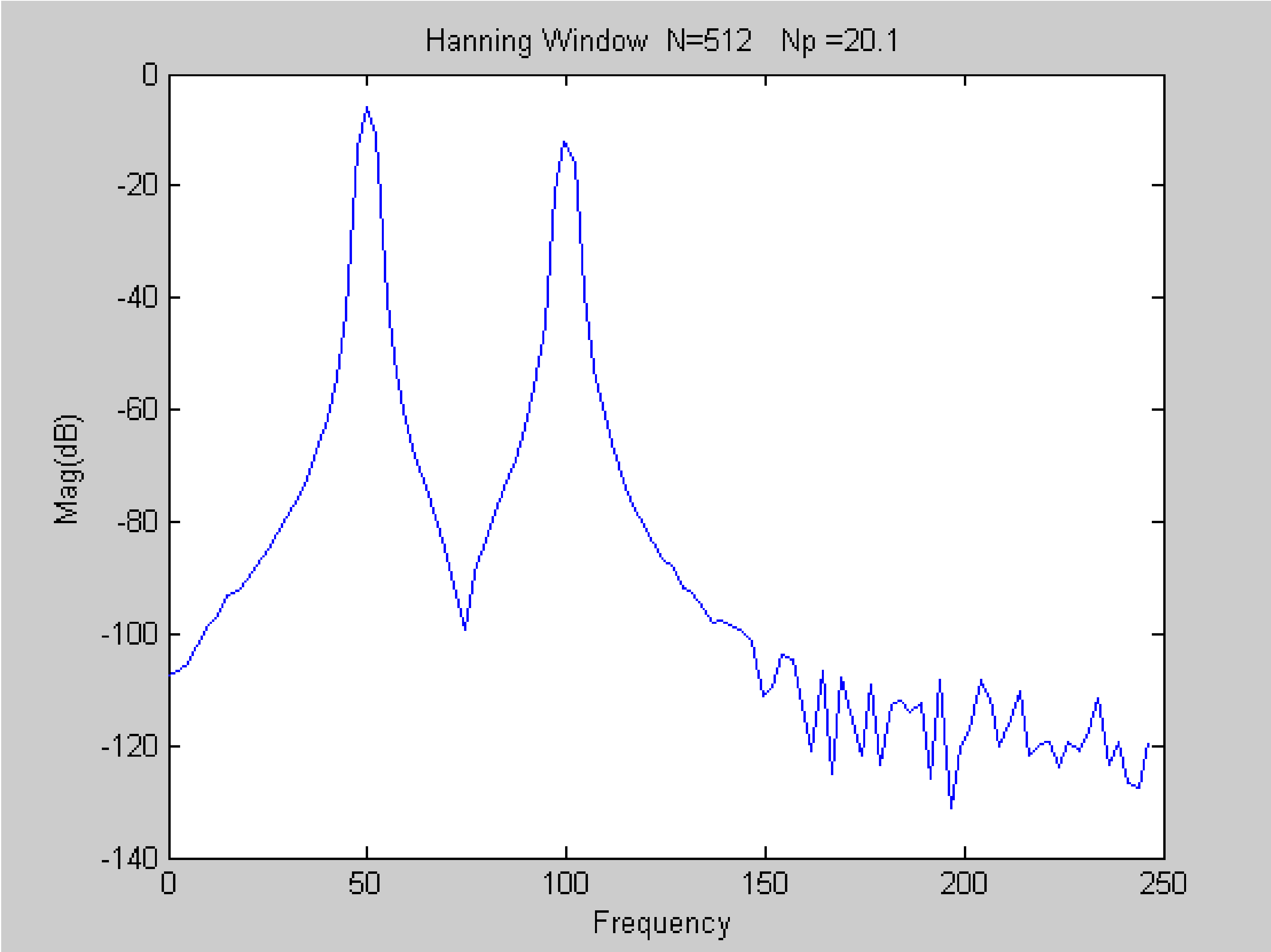
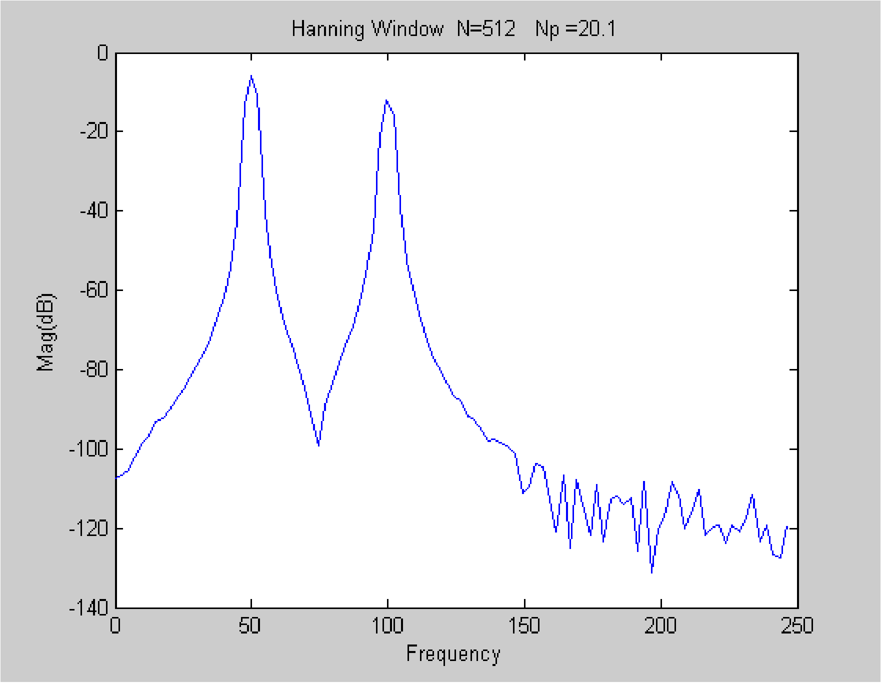
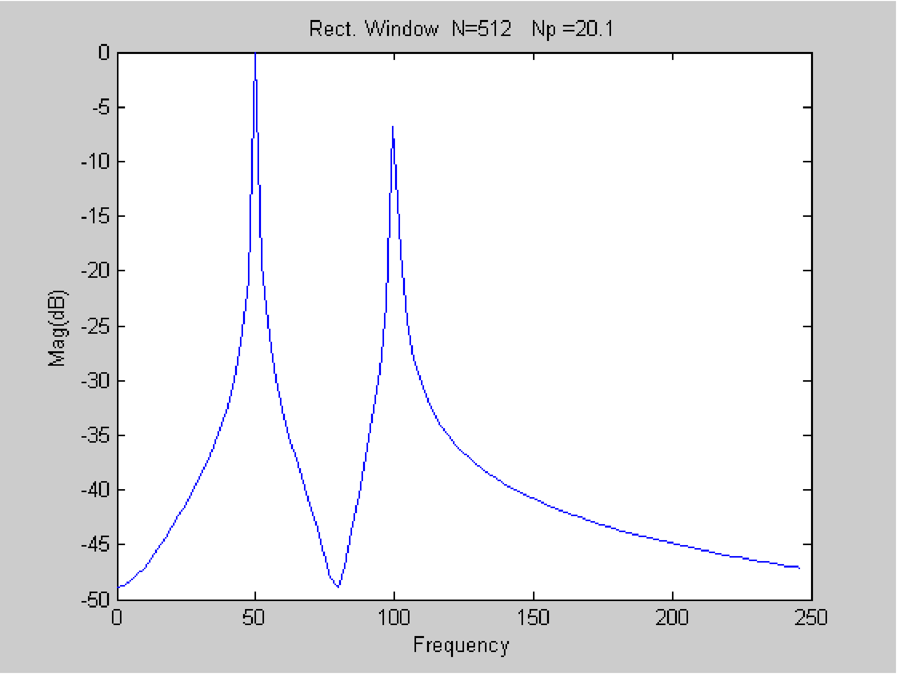


(zoomed in around fundamental)

Comparison with Rectangular Window





Note: Vertical axis are different

Hanning Window

Columns 1 through 7

-107.3123 -106.7939 -105.3421 -101.9488 -98.3043 -96.6522 -93.0343

Columns 8 through 14

-92.4519 -90.4372 -87.7977 -84.9554 -81.8956 -79.3520 -75.8944

Columns 15 through 21

-72.0479 -67.4602 -61.7543 -54.2042 -42.9597 -13.4511 -6.0601

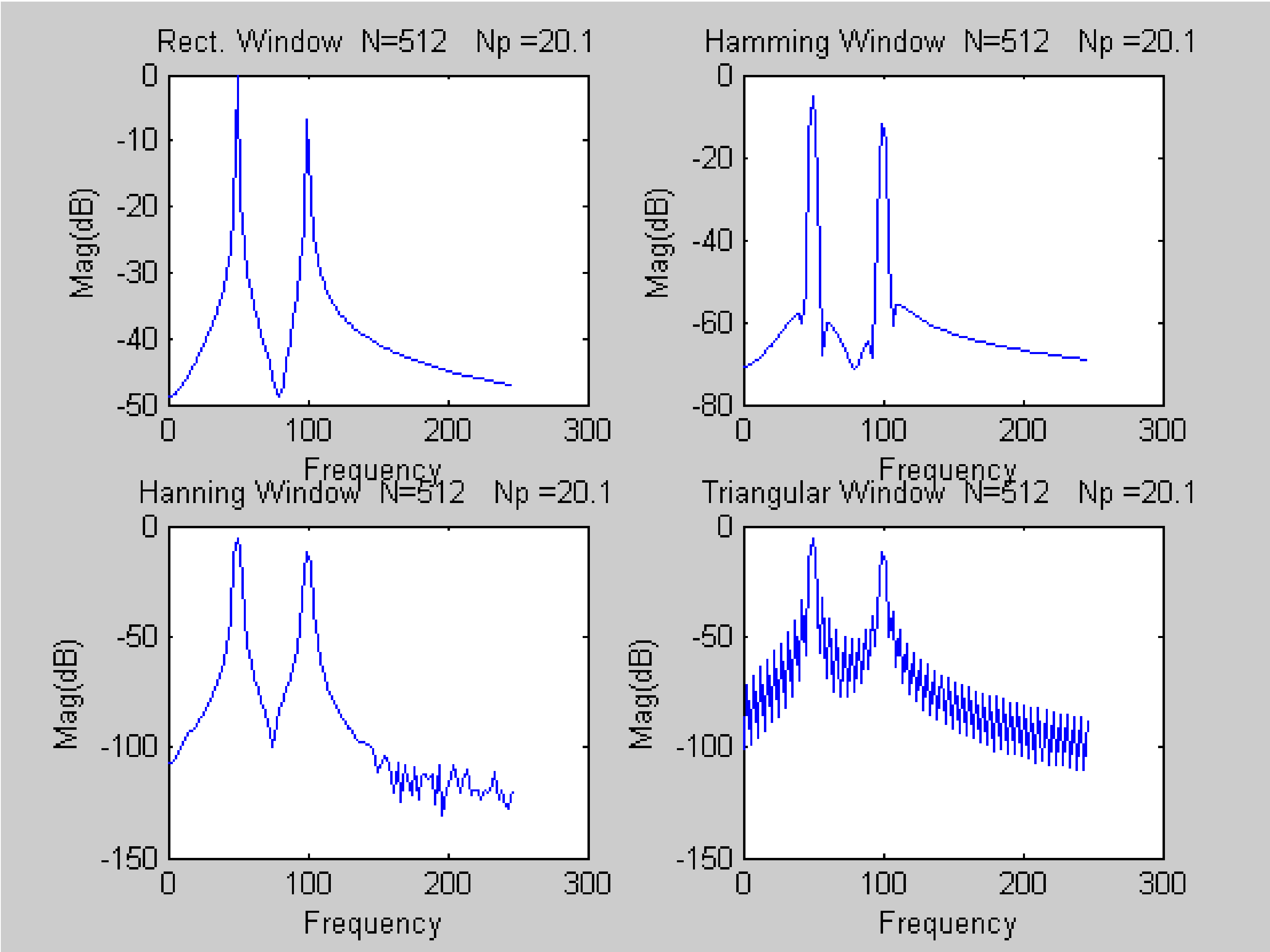
Columns 22 through 28

-10.8267 -40.4480 -53.3906 -61.8561 -68.3601 -73.9966 -79.0757

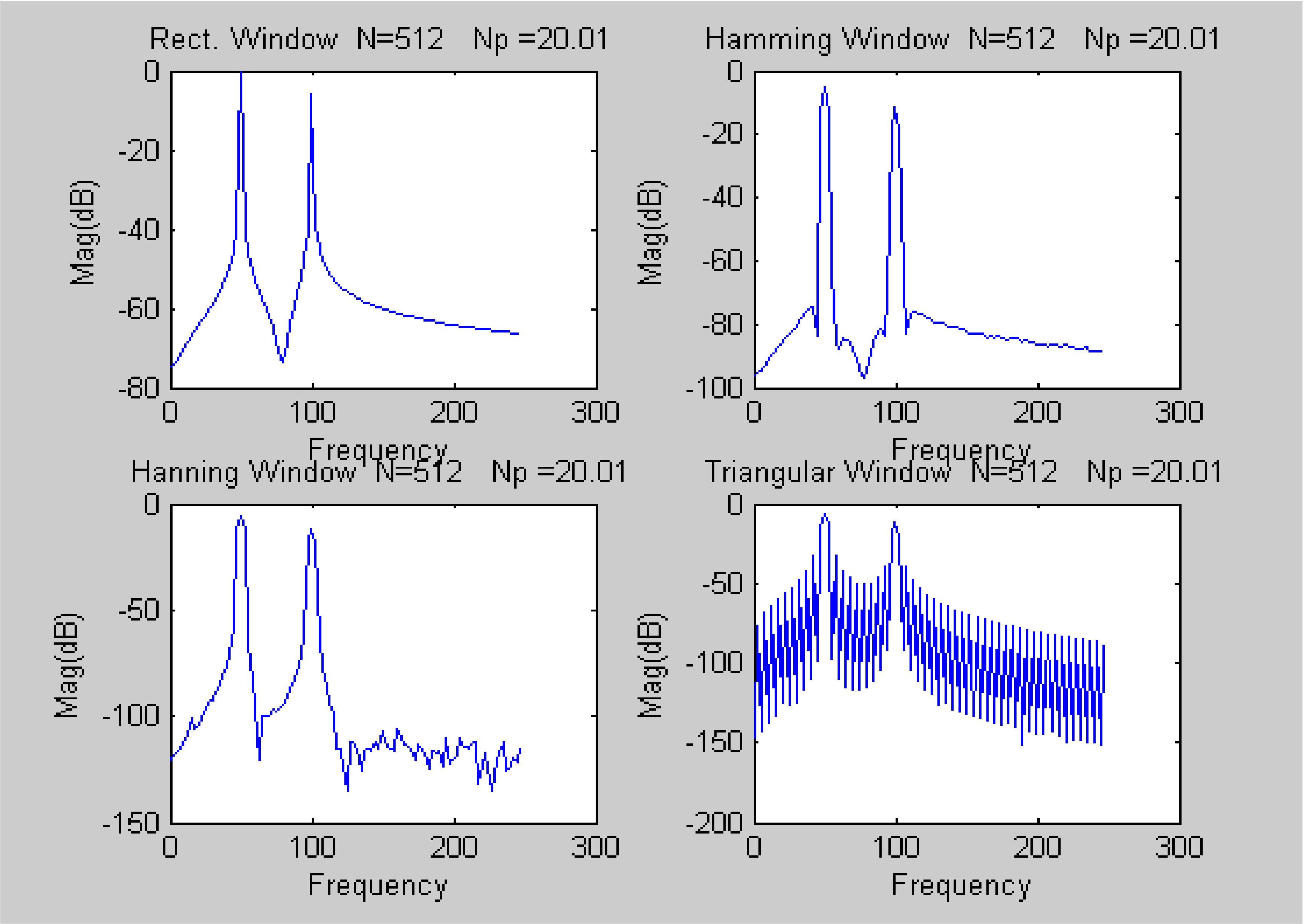
Columns 29 through 35

-84.4318 -92.7280 -99.4046 -89.0799 -83.4211 -78.5955 -73.9788

Comparison of 4 windows



Comparison of 4 windows

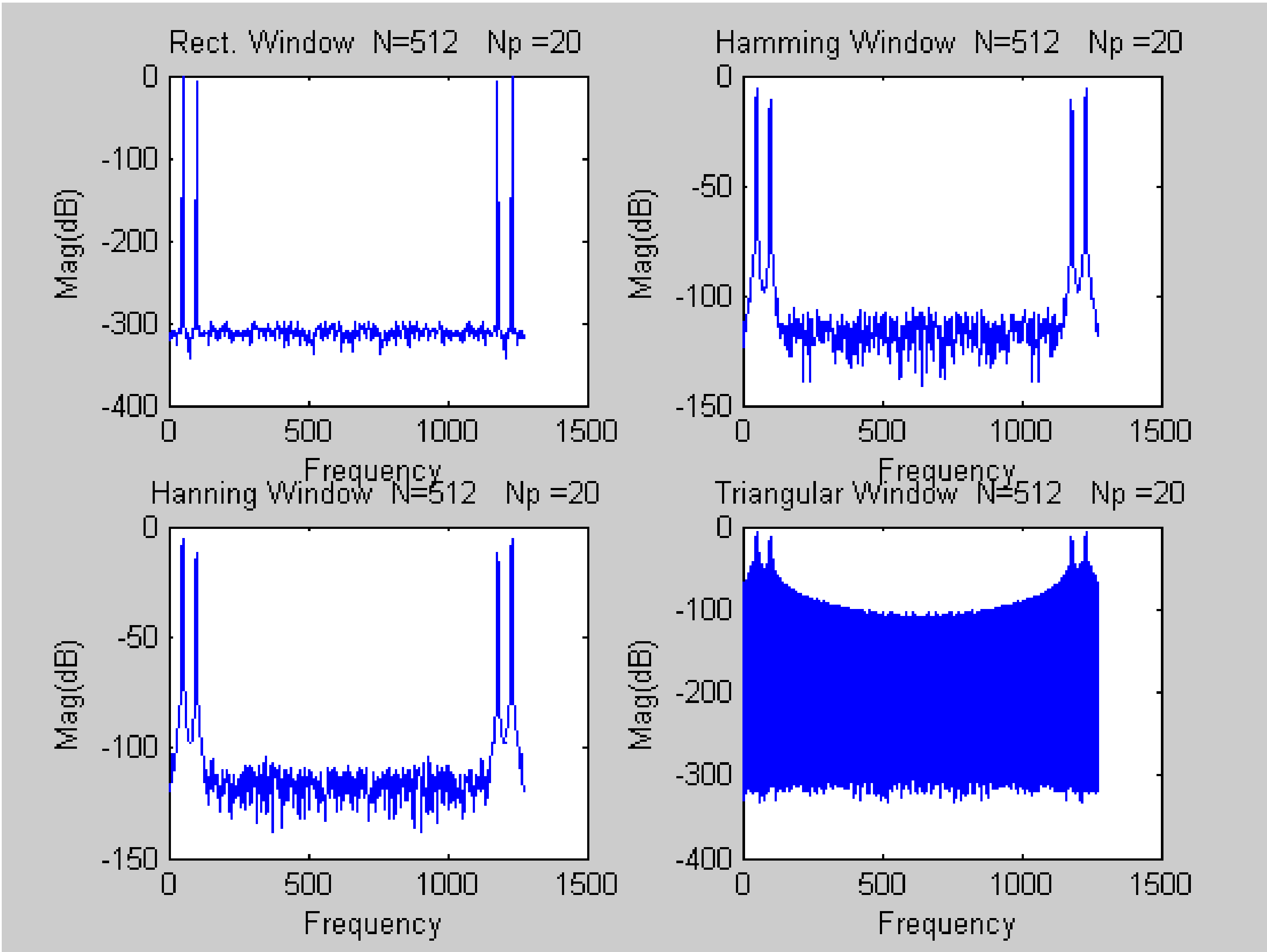


But windows can make things worse too!

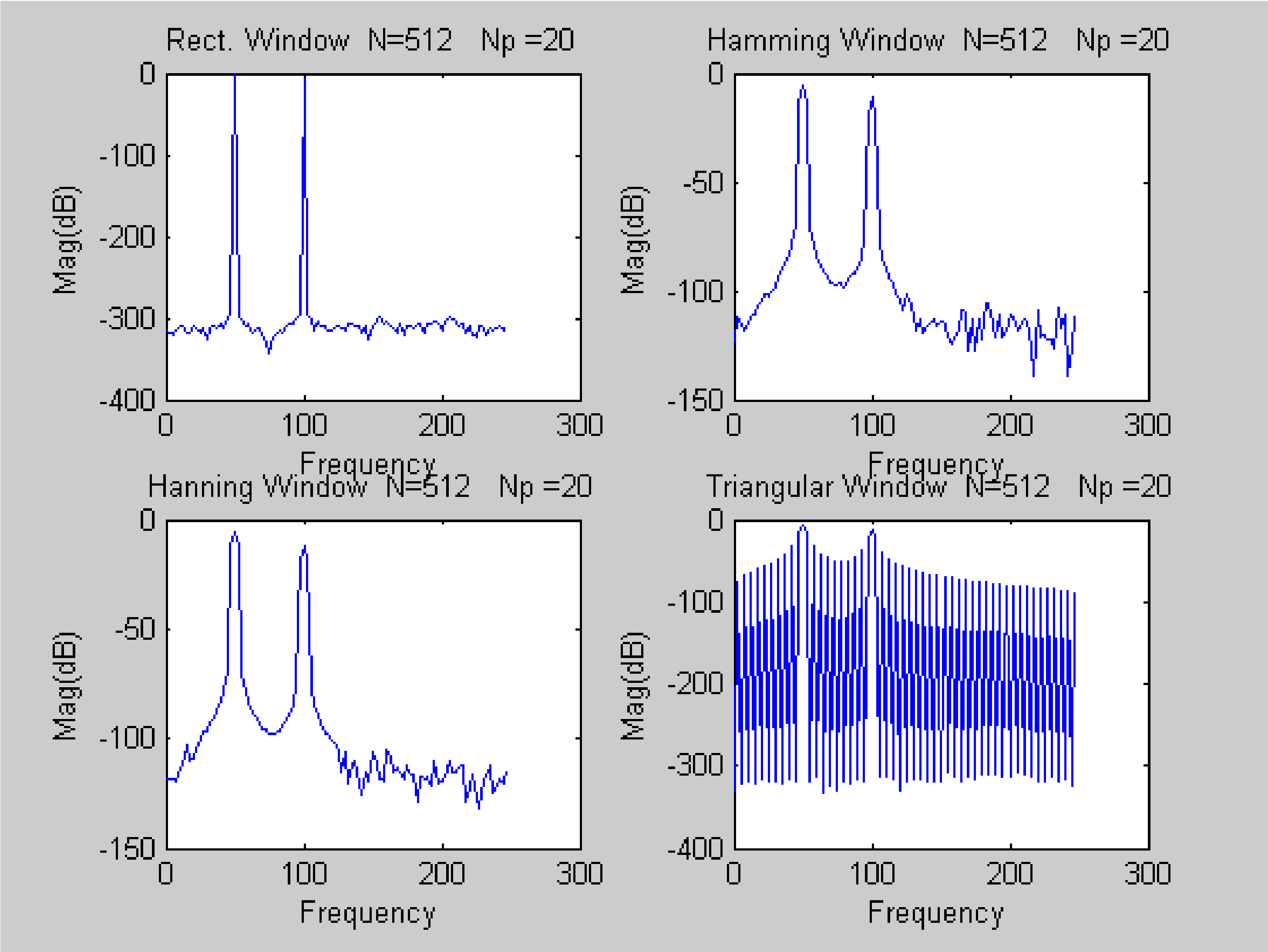
Consider situation where we really do have coherent sampling and a window is applied

fsig1=50Hz
fsig2=100Hz
N=512
Np=20

Comparison of 4 windows when sampling hypothesis are satisfied



Comparison of 4 windows



But windows can make things worse too!

Consider situation where we really do have coherent sampling and a window is applied

fsig1=50Hz
fsig2=100Hz
N=512
Np=20

And we do not really know how much worse thing can be!

Be careful about interpreting results obtained by using windowing to mitigate the non-coherent sampling problem !

Remember the hypothesis of the theorem relating the DFT, which is easy to calculate, to the Fourier Series coefficients!

Preliminary Observations about Windows

- Provide separation of spectral components
- Energy can be accumulated around spectral components
- Simple to apply
- Some windows work much better than others

But – windows do not provide dramatic improvement and can significantly degrade performance if sampling hypothesis are met

Issues of Concern for Spectral Analysis

An integral number of periods is critical for spectral analysis

Not easy to satisfy this requirement in the laboratory

Windowing can help but can hurt as well

Out of band energy can be reflected back into bands of interest

Characterization of CAD tool environment is essential

Spectral Characterization of high-resolution data converters requires particularly critical consideration to avoid simulations or measurements from masking real performance

Summary of time and amplitude quantization assessment

Time and amplitude quantization do not introduce <u>harmonic</u> distortion

Time and amplitude quantization do increase the noise floor

Performance Characterization of Data Converters

- Static characteristics
  - Resolution
  - Least Significant Bit (LSB)
  - Offset and Gain Errors
  - Absolute Accuracy
  - Relative Accuracy
  - Integral Nonlinearity (INL)
  - Differential Nonlinearity (DNL)
  - Monotonicity (DAC)
  - Missing Codes (ADC)
  - Quantization Noise
  - Low-f Spurious Free Dynamic Range (SFDR)
  - Low-f Total Harmonic Distortion (THD)
  - Effective Number of Bits (ENOB)
  - Power Dissipation

Quantization Noise

- DACs and ADCs generally quantize both amplitude and time
- If converting a continuous-time signal (ADC) or generating a desired continuous-time signal (DAC) these quantizations cause a difference in time and amplitude from the desired signal – this difference is termed “noise”.
- First a few comments about Noise

What is Noise in a data converter?

Noise is a term applied to some nonideal effects of a data converter

Precise definition of noise is probably not useful

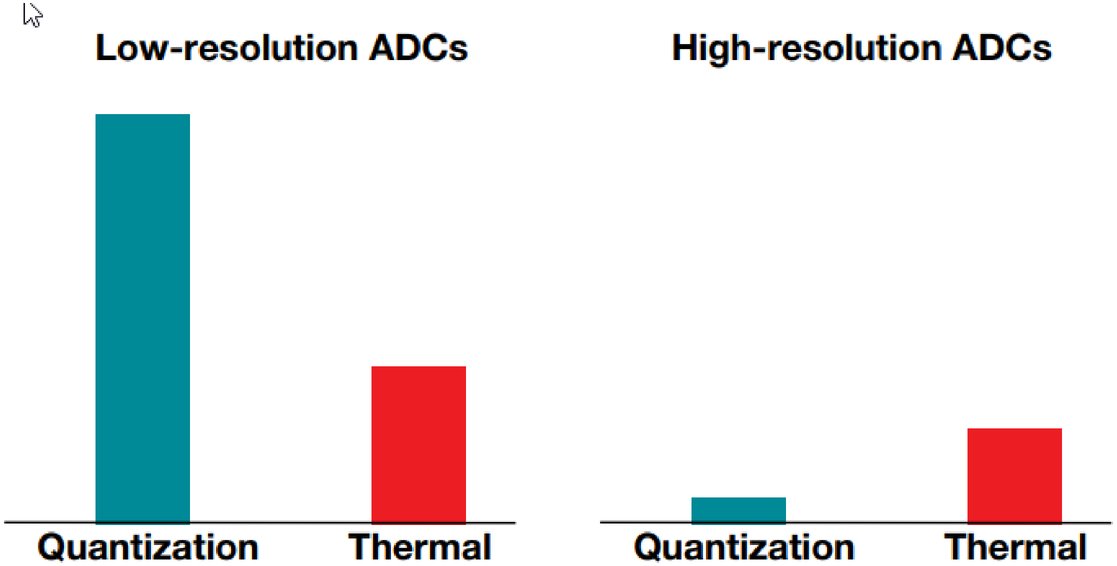
Some differences in views about what nonideal characteristics of a data converter should be referred to as noise

Types of noise:

- Random noise due to movement of electrons in electronic circuits (resistors and active devices) – highly dependent upon temperature thus often termed “thermal” noise
- Interfering signals generated by other systems
- Interfering signals generated by a circuit or system itself
- Error signals associated with imperfect signal processing algorithms or circuits
  - Quantization noise
  - Sample Jitter
  - Harmonic Distortion

Noise

Noise is any undesired signal (typically random) that adds to the desired signal, causing it to deviate from its original value.[1]



[1] from Bryan Lizon

[1]Definitiion from "Fundamentals of Precision ADC Noise Analysis", Bryan Lizon, Texas Instruments, Sept. 2020

Noise

Good reference on noise in ADCs

Fundamentals of Precision ADC Noise Analysis

Design tips and tricks to reduce noise with delta-sigma ADCs

TEXAS INSTRUMENTS

ti.com/precisionADC September | 2020 by Bryan Lizon

file:///C:/Users/rlgeiger/Documents/ABIN/Research/Noise/slyy192.pdf

Stay Safe and Stay Healthy !

End of Lecture 29